Motors and Power Transmission

ME72: Mechanical Engineering Design Laboratory
Sources of Mechanical Energy

- Potential Energy
  \[ E = mgh \]

- Kinetic Energy
  \[ E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]

- Spring Energy
  \[ E = \frac{1}{2}kx^2 \]

- Electrical Energy + Motor
  \[ E = VIt = \tau\omega t \]
Batteries

- Types of Batteries
  - Alkaline
  - Ni-Cd
  - NiMH
  - Lead-Acid
  - Lithium

- Battery Properties
  - Rechargeability
  - Energy Density
  - Capacity
  - Voltage
  - Internal Resistance
  - Discharge Rate
  - Shelf Life

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<table>
<thead>
<tr>
<th>Battery Chemistry</th>
<th>Recharge</th>
<th>Energy Density (W/hr/kg)</th>
<th>Cell Voltage</th>
<th>Typical Capacity (mAh)</th>
<th>Internal Resistance (ohms)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alkaline</td>
<td>No</td>
<td>130</td>
<td>1.5</td>
<td>AA 1400</td>
<td>C 4500</td>
<td>D 10000</td>
</tr>
<tr>
<td>Lead-Acid</td>
<td>Yes</td>
<td>40</td>
<td>2.0</td>
<td>1.2 - 120 Ah</td>
<td>C-size 0.006</td>
<td>Available in a wide variety of sizes</td>
</tr>
<tr>
<td>Lithium</td>
<td>No</td>
<td>300</td>
<td>3.0</td>
<td>A 1800</td>
<td>C 5000</td>
<td>D 14000</td>
</tr>
<tr>
<td>Mercury</td>
<td>No</td>
<td>120</td>
<td>1.35</td>
<td>Coin 190</td>
<td>10</td>
<td>Low internal resistance, available from many sources</td>
</tr>
<tr>
<td>NiCd</td>
<td>Yes</td>
<td>38</td>
<td>1.2</td>
<td>AA 500</td>
<td>C 1800</td>
<td>D 4000</td>
</tr>
<tr>
<td>NiMH</td>
<td>Yes</td>
<td>57</td>
<td>1.3</td>
<td>AA 1800</td>
<td>C 4000</td>
<td>4/3A 2300</td>
</tr>
<tr>
<td>Silver</td>
<td>No</td>
<td>130</td>
<td>1.6</td>
<td>Coin 180</td>
<td>10</td>
<td>High energy density but not widely available, limited range of sizes</td>
</tr>
<tr>
<td>Zinc-Air</td>
<td>No</td>
<td>310</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon-Zinc</td>
<td>No</td>
<td>75</td>
<td>1.5</td>
<td>D 6000</td>
<td></td>
<td>Inexpensive but obsolete</td>
</tr>
</tbody>
</table>

All numbers listed here are approximate. Precise values depend on the details of the particular battery. Some values depend on the battery's state of charge, temperature, and discharge history.

**Figure 8.1:** Comparison of characteristics for selected batteries and sizes.
Battery Discharge Curves

• The graph is normalized with respect to a lithium battery
• The dashed lines show output voltage versus battery capacity consumed
• The solid lines show voltage versus time
DC Motor Model

Physical Principles
- Current through a wire produces a magnetic field
- A wire moving through a magnetic field induces a current
- For multiple windings we find that

\[ \tau = K_t i \]

Motor electrical model

\[ L_a \frac{di_a}{dt} + R_m i + K_e \omega = V_a \]

Substituting in for \( i \) and solving for \( \tau \) gives

\[ \tau = \frac{K_t}{R_m} V_a - \frac{K_t K_e}{R_m} \omega \]
Torque-Speed Curves

- DC Motor Equation

\[ \tau = \frac{K_t}{R_m} V_a - \frac{K_t K_e}{R_m} \omega \]

- At \( \omega = 0 \)

\[ \tau = \frac{K_t}{R_m} V_a = \tau_{stall} \]

- At \( \tau = 0 \)

\[ \omega = \frac{V_a}{K_e} = \omega_{noload} \]

Torque-Speed Equation

\[ \tau = \tau_{stall} - \frac{\tau_{stall}}{\omega_{noload}} \omega \]

A transmission changes the slope of the torque-speed curve (line) to provide more desirable no load speed and output torque characteristics.
## Motor Specifications

**D.C. motor escap® 16 M11**

<table>
<thead>
<tr>
<th>Standard types available from stock</th>
<th>-210</th>
<th>-208</th>
<th>-207</th>
<th>-205</th>
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</thead>
<tbody>
<tr>
<td>Measuring voltage</td>
<td>V</td>
<td>6</td>
<td>7.5</td>
<td>9</td>
</tr>
<tr>
<td>No-load speed</td>
<td>rpm</td>
<td>8400</td>
<td>7800</td>
<td>8300</td>
</tr>
<tr>
<td>Stall torque</td>
<td>mNm</td>
<td>3</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>oz-in</td>
<td>0.42</td>
<td>0.35</td>
<td>0.33</td>
</tr>
<tr>
<td>Power output</td>
<td>W</td>
<td>0.7</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Av. no-load current</td>
<td>mA</td>
<td>7</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Typical starting voltage</td>
<td>V</td>
<td>0.06</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Max. continuous current</td>
<td>A</td>
<td>0.4</td>
<td>0.28</td>
<td>0.24</td>
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<tr>
<td>Max. recommended speed</td>
<td>rpm</td>
<td>12000</td>
<td>12000</td>
<td>12000</td>
</tr>
<tr>
<td>Max. angular acceleration</td>
<td>$10^3 \text{ rad/s}^2$</td>
<td>96</td>
<td>114</td>
<td>120</td>
</tr>
<tr>
<td>Back-EMF constant</td>
<td>V/1000 rpm</td>
<td>0.7</td>
<td>0.94</td>
<td>1.1</td>
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<tr>
<td>Rotor inductance</td>
<td>mH</td>
<td>0.5</td>
<td>0.8</td>
<td>1</td>
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<tr>
<td>Motor regulation R/k²</td>
<td>$10^3$/Nms</td>
<td>300</td>
<td>330</td>
<td>380</td>
</tr>
<tr>
<td>Terminal resistance</td>
<td>ohm</td>
<td>13.4</td>
<td>27</td>
<td>39.5</td>
</tr>
<tr>
<td>Torque constant</td>
<td>mNm/A</td>
<td>6.7</td>
<td>9</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>oz-in/A</td>
<td>0.949</td>
<td>1.28</td>
<td>1.44</td>
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<tr>
<td>Rotor inertia</td>
<td>kgm²·10⁻⁷</td>
<td>0.7</td>
<td>0.56</td>
<td>0.5</td>
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<tr>
<td>Mechanical time constant</td>
<td>ms</td>
<td>20</td>
<td>18</td>
<td>19</td>
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<tr>
<td>Thermal time constant</td>
<td>s</td>
<td>6</td>
<td>5</td>
<td>4</td>
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<tr>
<td>Thermal resistance</td>
<td>rotor-body</td>
<td>380</td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td></td>
<td>body-ambient</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>

- **Key Specs:** No-load speed, stall torque, torque constant.
- **For Design:** Peak and continuous current, continuous torque rating.
Motor Power

When does a motor operate at maximum power?

\[ P = \tau \omega = (\tau_{\text{stall}} - \frac{\tau_{\text{stall}}}{\omega_{\text{noload}}} \omega) \omega \]

To find the maximum operating point, take the derivative and set it equal to zero.

\[ \frac{\partial P}{\partial \omega} = \tau_{\text{stall}} - \frac{2\tau_{\text{stall}}}{\omega_{\text{noload}}} \omega = 0 \]

Solve for \( \omega^* \) and corresponding \( \tau^* \)

\[ \omega^* = \frac{\omega_{\text{noload}}}{2} \]

\[ \tau^* = \tau_{\text{stall}} - \frac{\tau_{\text{stall}}}{\omega_{\text{noload}}} \cdot \frac{\omega_{\text{noload}}}{2} = \frac{\tau_{\text{stall}}}{2} \]
Power and Efficiency

\[ I_{stall} = \frac{V_{in}}{R_m} \]

Efficiency: \[ \eta_{max} = \left( 1 - \sqrt{\frac{I_{no\text{load}}}{I_{stall}}} \right) \]
Battery Life Calculations

Typical Battery Specs

Voltage (V) = 1.5 V  Capacity (C) = 1000 mAh

Note that 1 Joule = 1 Coulomb-Volt and 1 Amp = 1 Coulomb/Sec

Battery Energy

\[ E = VC \frac{3600 \text{ s/h}}{1000 \text{ mA/A}} = 3.6VC \text{ Joules} \]

Battery Life

\[ t = \frac{E}{P} \]

Example: at maximum power, the motor requires

\[ P = 1.15 \text{ N-cm} (0.01 \text{ m/cm}) 2700 \text{ rpm} (2\pi/60 \text{ s}) = 3.25 \text{ N-m/s} = 3.25 \text{ J/s} = 3.25 \text{ W} \]

With 8 batteries operating at 1.2 V with 1000mAh ratings we have:

\[ \text{Life (hours)} = \frac{8(1.2)(1000)3.6}{3.25(3600)} = 2.9 \text{ hours} \]

With 4 motors, and other losses, you will be lucky to get 45 minutes of operation
Power Transmission: Motivation

What kind of useful motion can we get at 2000 to 2500 RPM?
Linear Velocity and Force

Basic Equations:

\[ v = 2\pi \omega r \]

\[ F = \tau / r \]

Example Calculation:

\[ \omega = 2500 \text{ RPM} \quad \tau_{max} = 3.26 \text{ oz-in} \quad r = 2 \text{ in} \]

\[ v = 2\pi (2/12)2500 = 2619 \text{ ft/min} \]

\[ v = 29.75 \text{ mph} \]

\[ F = 1.63 / 2 = .815 \text{ oz (at max power)} \]

\[ F = 3.26 / 2 = 1.63 \text{ oz (at stall)} \]

In order to get useful forces and motions at/near the maximum power/efficiency point of the motor, we need to make \( r \) smaller.

A transmission allows the designer to effectively reduce \( r \) while maintaining other design constraints.
Power Transmission: Basic Principle

\[
\tau_{in}\omega_{in} = \tau_{out}\omega_{out}
\]

\[
F_{in}v_{in} = F_{out}v_{out}
\]

Conservation of power(energy) allows us to make trade-offs between force and speed.
Transmission Ratio and Effective Radius

A transmission ratio is defined as:

\[ r_t = \frac{\omega_{out}}{\omega_{in}} = \frac{\tau_{in}}{\tau_{out}} \]

For a compound transmission (gears, belts, chains) \( r_t \) has the form:

\[ r_t = \pm \frac{\text{product of driver radii}}{\text{product of driven radii}} \]

If we are interested in a particular output velocity or force, we have:

\[ V_{out} = r_w \omega_{out} = r_w r_t \omega_{in} = r_e \omega_{in} \]

\[ F_{out} = \frac{\tau_{out}}{r_w} = \frac{\tau_{in}}{r_t r_w} = \frac{\tau_{in}}{r_e} \]

\( r_e \) is called the effective radius.
For a wheeled vehicle, the force available for pushing comes from friction.

Force balance:

\[ F_{app} = \mu mg \cos(\theta) - mg \sin(\theta) \]

On a flat surface this is simply:

\[ F_{app} = \mu mg \]

Since the motor is the driving force we can write:

\[ F_{app} = \mu mg \geq \frac{\tau_m}{rt wr} \]

This means

\[ re = rt wr \geq \frac{\tau_m}{\mu mg} \]

otherwise slipping will occur (on a flat surface).
Lifting

How big do we make a wheel/pulley to lift the most weight at a fixed velocity?

Start with the motor equation:

$$\tau = \tau_{st} - \frac{\tau_{st} \omega}{\omega_{nl}}$$

Substitute in $\tau = W/r$ and $\omega := v/r$ and solve for $W$:

$$W = \frac{\tau_{st}}{r} - \frac{\tau_{st} v}{\omega_{nl} r^2}$$

Maximize $W$ by taking the partial w.r.t $r$ and setting it to zero.

$$\frac{\partial W}{\partial r} = -\frac{\tau_{st}}{r^2} + \frac{2\tau_{st} v}{\omega_{nl} r^3} = 0$$

Solve for optimum $r^*$ and $W^*$:

$$r^* = \frac{2v}{\omega_{nl}} \quad W^* = \frac{\tau_{st} \omega_{nl}}{4v}$$
Transmission Design

1. Determine force and velocity requirements
2. Match motor to power requirements
3. Determine output wheel/pulley size limitations
4. Determine transmission ratio requirements
5. Select transmission elements based on power, ratio, and space requirements
6. Design transmission housing to support loads, minimize deflections, maintain alignments, etc.

Note: It is not unusual for items 1 and 2 to be reversed, that is, force and velocity requirements often come from pre-existing power specifications.
References