Sources of Mechanical Energy

- Potential Energy
  \[ E = mgh \]

- Kinetic Energy
  \[ E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \]

- Spring Energy
  \[ E = \frac{1}{2}kx^2 \]

- Electrical Energy + Motor
  \[ E = VIt = \tau\omega t \]
Batteries

- Types of Batteries
  - Alkaline
  - Ni-Cd
  - NiMH
  - Lead-Acid
  - Lithium

- Battery Properties
  - Rechargeability
  - Energy Density
  - Capacity
  - Voltage
  - Internal Resistance
  - Discharge Rate
  - Shelf Life

<table>
<thead>
<tr>
<th>Battery Chemistry</th>
<th>Recharge</th>
<th>Energy Density (Whr/kg)</th>
<th>Cell Voltage</th>
<th>Typical Capacity (mAh)</th>
<th>Internal Resistance (ohms)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alkaline</td>
<td>No</td>
<td>130</td>
<td>1.5</td>
<td>AA 1400</td>
<td>C 4500 0.1</td>
<td>Most common primary battery</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>D 10000</td>
<td></td>
</tr>
<tr>
<td>Lead-Acid</td>
<td>Yes</td>
<td>40</td>
<td>2.0</td>
<td>1.2 - 120 A/h C-size 0.006</td>
<td></td>
<td>Available in a wide variety of sizes</td>
</tr>
<tr>
<td>Lithium</td>
<td>No</td>
<td>300</td>
<td>3.0</td>
<td>A 1800</td>
<td>C 5000 0.3</td>
<td>Excellent energy density, high unit cost</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>D 14000</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>No</td>
<td>120</td>
<td>1.35</td>
<td>Coin 190</td>
<td>10</td>
<td>Low internal resistance, available from many sources</td>
</tr>
<tr>
<td>NiCd</td>
<td>Yes</td>
<td>38</td>
<td>1.2</td>
<td>AA 500</td>
<td>C 1800 0.009</td>
<td>Available in a wide variety of sizes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>D 4000</td>
<td></td>
</tr>
<tr>
<td>NiMH</td>
<td>Yes</td>
<td>57</td>
<td>1.3</td>
<td>AA 1100</td>
<td>4/3A 2300</td>
<td>Better energy density than NiCd, expensive</td>
</tr>
<tr>
<td>Silver</td>
<td>No</td>
<td>130</td>
<td>1.6</td>
<td>Coin 180</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Zinc-Air</td>
<td>No</td>
<td>310</td>
<td>1.4</td>
<td></td>
<td></td>
<td>High energy density but not widely available, limited range of sizes</td>
</tr>
<tr>
<td>Carbon-Zinc</td>
<td>No</td>
<td>75</td>
<td>1.5</td>
<td>D 6000</td>
<td></td>
<td>Inexpensive but obsolete</td>
</tr>
</tbody>
</table>

All numbers listed are approximate. Precise values depend on the details of the particular battery. Some values depend on the battery's state of charge, temperature, and discharge history.

Figure 8.1: Comparison of characteristics for selected batteries and sizes.
Battery Discharge Curves

- The graph is normalized with respect to a lithium battery
- The dashed lines show output voltage versus battery capacity consumed
- The solid lines show voltage versus time
DC Motor Model

Physical Principles
- Current through a wire produces a magnetic field
- A wire moving through a magnetic field induces a current
- For multiple windings we find that

\[ \tau = K_t i \]

Motor electrical model

\[ L_a \frac{di_a}{dt} + R_m i + K_e \omega = V_a \]

Substituting in for \( i \) and solving for \( \tau \) gives

\[ \tau = \frac{K_t V_a}{R_m} - \frac{K_t K_e}{R_m} \omega \]
Torque-Speed Curves

- DC Motor Equation
  \[ \tau = \frac{K_t}{R_m} V_a - \frac{K_t K_e}{R_m} \omega \]

- At \( \omega = 0 \)
  \[ \tau = \frac{K_t}{R_m} V_a = \tau_{\text{stall}} \]

- At \( \tau = 0 \)
  \[ \omega = \frac{V_a}{K_e} = \omega_{\text{noload}} \]

Torque-Speed Equation
\[ \tau = \tau_{\text{stall}} - \frac{\tau_{\text{stall}}}{\omega_{\text{noload}}} \omega \]

A transmission changes the slope of the torque-speed curve (line) to provide more desirable no load speed and output torque characteristics.
Motor Power

When does a motor operate at maximum power?

\[ P = \tau \omega = (\tau_{\text{stall}} - \frac{\tau_{\text{stall}}}{\omega_{\text{noload}}}) \omega \]

To find the maximum operating point, take the derivative and set it equal to zero.

\[ \frac{\partial P}{\partial \omega} = \tau_{\text{stall}} - \frac{2\tau_{\text{stall}}}{\omega_{\text{noload}}} \omega = 0 \]

Solve for \( \omega^* \) and corresponding \( \tau^* \)

\[ \omega^* = \frac{\omega_{\text{noload}}}{2} \]

\[ \tau^* = \tau_{\text{stall}} - \frac{\tau_{\text{stall}} \omega_{\text{noload}}}{2} = \frac{\tau_{\text{stall}}}{2} \]
Power and Efficiency

\[ I_{\text{stall}} = \frac{V_{\text{in}}}{R_m} \]

Efficiency: \[ \eta_{\text{max}} = \left(1 - \sqrt{\frac{I_{\text{no load}}}{I_{\text{stall}}}}\right) \]