Machine Components: Power Transmission Elements

ME 72 Engineering Design Laboratory

Power Transmission Design Issues

- Power source (motor, engine, etc.)
- Power required \( (P = \text{torque} \times \text{velocity}) \)
- Continuous or Intermittent Motion
- Operating conditions (start, load duration)
- Magnitude of speed (input or output)
- Speed modification (output to input, constant or variable, linear or rotary)
Constant Speed Mechanical Transmission Elements

- Gears
- Friction wheels
- Power Screws
- Belts and Chains
- Wire Rope
- Couplings
- Clutches and Brakes
Relative Shaft Position

1. Collinear
   - Couplings
   - Clutches

2. Parallel
   - (Center distance)
   - (Large)
   - Belts
   - Chains
   - Ropes
   - Universal joints
   - Four-bar mechanisms

3. Intersecting
   - Bevel gears
   - Universal joints
   - Friction drives
   - (Small)
   - Spur gears
   - Helical gears
   - Friction drives
   - Universal joints

4. Nonintersecting
   - Crossed helical gears
   - Slew bevel gears
   - Hypoid gears
   - Worm gearing
   - Flexible shaft
   - Friction drives

Relative Size

- Flat-belt drive
- Roller chain drive
- V-belt drive
- Gear reducer

1000 mm
Gear Design Issues

- Types of Gears: spur, helical, herringbone, internal, rack, bevel, and worm
- Angular Velocity Ratio (gear ratio)
- Power Requirements (speed and torque)
- Gear Tooth Loads
- Gear Tooth Stresses
Gear Terminology

- Pitch Circle (radius or diameter)
- Circular Pitch
- Number of Teeth
- Pressure Angle
- Face Width
Spur Gears: Force Analysis

- $W$ is the resultant force
- $W_t$ and $W_r$ are tangential and radial components
- $d$ is the pitch diameter
- $\phi$ is the pressure angle
- $\tau$ is the applied torque

\[
W_t = \frac{2\tau}{d} \\
W = \frac{W_t}{\cos \phi} \\
W_r = W_t \tan \phi
\]

Helical Gears: Force Analysis

- $W_t$, $W_r$, and $W_a$ are tangential and radial and axial components
- $\phi_n$ is the normal pressure angle
- $\phi_t$ is the tangent pressure angle
- $\psi$ is the helix angle

\[
W_t = \frac{2\tau}{d} \\
W_r = W \sin \phi_n = W_t \tan \phi_t \\
W_t = W \cos \phi_n \cos \psi \\
W_a = W \cos \phi_n \sin \psi = W_t \tan \psi
\]
Bevel Gears: Force Analysis

- $W_t$, $W_r$ and $W_a$ are tangential and radial and axial components
- $\phi$ is the pressure angle
- $\gamma$ is the bevel angle
- $r_{av}$ is the average pitch radius

\[
W_t = \frac{\tau}{r_{av}}
\]
\[
W_r = W_t \tan \phi \cos \gamma
\]
\[
W_a = W_t \tan \phi \sin \gamma
\]

Worm Gears: Force Analysis

- $W_t$, $W_r$ and $W_a$ are tangential and radial and axial components
- $\phi_n$ is the normal pressure angle
- $\lambda$ is the worm lead angle
- $d_{\text{worm}}$ is the pitch diameter of the worm

\[
W_t = \frac{2\tau}{d_{\text{worm}}}
\]
\[
W^x = W_t = W \cos \phi_n \sin \lambda
\]
\[
W^y = W_r = W \sin \phi_n
\]
\[
W^z = W_a = W \cos \phi_n \cos \lambda
\]
Stresses in Gears: Lewis Formula

\[
\sigma_b = \frac{M_c}{I} = \frac{W_l(l/2)}{b_s r^2} = \frac{6W_l}{b_s t^2}
\]

\[
l = \frac{t^2}{4x}
\]

\[
\sigma_b = \frac{3W}{2b_w x} = \frac{3W_p d}{2b_w p_d x} = \frac{W_p d}{b_w Y}
\]

\[
Y = \frac{2p_d x}{3} = \text{Lewis Form Factor}
\]

Stresses in Gears: AGMA Formula

- **Assumptions**
  - Contact ratio is 1-2
  - No interference or undercutting
  - No pointed teeth
  - Nonzero backlash
  - Standard root fillets
  - Friction neglected
  - \(\sigma_b\)=bending stress
  - \(W\)=tangential force
  - \(p_d\)=pitch diameter
  - \(b_w\)=face width
  - \(J\)=geometry factor
  - \(K_a\)=application factor
  - \(K_m\)=load distribution factor
  - \(K_s\)=size factor
  - \(K_v\)=dynamic factor

\[
\sigma_b = \frac{W_l p_d}{b_w J} \frac{K_a K_m K_s}{K_v}
\]
Power Screws

- Consists of a threaded shaft and nut
- Used to convert rotary to linear motion
- Can be designed for high precision
- Often have multiple threads
- Non backdrivable

Power Screw Force Balance

- Basic force balance

\[
\sum F_y = F - \mu N \cos \lambda - N \sin \lambda = 0
\]
\[
F = N(\mu \cos \lambda + \sin \lambda)
\]
\[
\sum F_y = N \cos \lambda - \mu N \sin \lambda = 0
\]
\[
N = \frac{P}{(\cos \lambda - \mu \sin \lambda)}
\]
\[
F = P \left( \frac{\mu \cos \lambda + \sin \lambda}{\cos \lambda - \mu \sin \lambda} \right)
\]
\[
T = \frac{P d_p}{2} \left( \frac{\mu \pi d_p + L}{\pi d_p - \mu L} \right)
\]
- Rewrite in terms of \( L \) using:

\[
\tan \lambda = \frac{L}{\pi d_p}
\]

\[
T_{up} = \frac{P d_p}{2} \left( \frac{\mu \pi d_p + L}{\pi d_p - \mu L} \right)
\]
\[
T_{down} = \frac{P d_p}{2} \left( \frac{\mu \pi d_p - L}{\pi d_p + \mu L} \right)
\]
Belt Drives

- Belt drives are used when large distances between shafts make gears impractical or when designated speed is too high for chain drives.
- Belts are used with pulleys or sheaves to transmit power.
- Belts require tensioning, and are prone to slip under high loads.

Types of Belts

![Types of Belts](image)

*Figure 11-6* Types of belts. (Courtesy Deere and Company Technical Services.)
Chain Drives

- Chains transmit power through interlocking links wrapping on a sprocket.
- Chain drives have a high load capacity.
- They can be used to transmit power or impart timed linear motion.

Types of Chains
Chain Attachments

Belt/Chain Length

\[ \overline{AB}^2 = c_d^2 - \left( \frac{D_2 - D_1}{2} \right)^2 \]

\[ AB = \frac{1}{2} \sqrt{c_d^2 - (D_2 - D_1)^2} \]

\[ \phi_1 = \pi - 2\alpha \]

\[ \phi_2 = \pi + 2\alpha \]

\[ \alpha = \sin^{-1} \left( \frac{D_2 - D_1}{2c_d} \right) \]

\[ L = 2\overline{AB} + \frac{D_1}{2} \phi_1 + \frac{D_2}{2} \phi_2 \]
Wire Rope

- Wire rope can also be used to transmit power.
- It is characterized by the number of bundles and wires/bundle.
- The critical design parameter is the pulley to rope diameter ratio.

Summary

- Gears transmit power between a variety of intersecting and non-intersecting shafts.
- Gear forces and tooth stresses are key design parameters for gear trains.
- Power screws convert rotary to linear motion and produce high forces from small torques.
- Belts and Chains transmit power through larger distances than gears.
- Wire rope (or cable) can provide backlash-free power transmission for a limited range of motion.
References