Imprecision in Engineering Design *†

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Abstract

Methods for incorporating imprecision in engineering design decision-making are briefly reviewed and compared. A tutorial is presented on the Method of Imprecision (MoI), a formal method, based on the mathematics of fuzzy sets, for representing and manipulating imprecision in engineering design. The results of a design cost estimation example, utilizing a new informal cost specification, are presented. The MoI can provide formal information upon which to base decisions during preliminary engineering design and can facilitate set-based concurrent design.

Introduction

One of the most critical problems in engineering design is making early decisions on a sound basis. However, the early stages of design are also the most uncertain, and obtaining precise information upon which to base decisions is usually impossible. The primary reason for this difficulty is that imprecision is an integral part of the engineering design process. Not imprecision in thought or logic, but rather the intrinsic vagueness of a preliminary, incomplete description. At the concept stage, the design description is nearly completely vague or imprecise (fuzzy). The design process reduces this imprecision until ultimately the final description is precise (crisp), except for tolerances, which represent the allowable limits on stochastic manufacturing variations.

Despite this evolution of imprecision, engineering design methods and computer aids have nearly all utilized precise information (though some can include stochastic effects).

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Solid modeling CAD systems, for example, require precise geometry; there is no option to indicate that a dimension is imprecise or only vaguely known.[1]

The need for a methodology to represent and manipulate imprecision is greatest in the early, preliminary phases of engineering design, where the designer is most unsure of the final dimensions and shape, materials and properties, and performance of the completed design. Additionally, the most important decisions, those with the greatest effect on overall cost, are made in these early stages [35, 89, 103, 96].

> "If a major project is truly innovative, you cannot possibly know its exact cost and its exact schedule at the beginning. And if in fact you do know the exact cost and the exact schedule, chances are that the technology is obsolete." [27][2]

This paper will review imprecision and uncertainty methods in engineering design, then present a brief tutorial of the *Method of Imprecision* (MoI), followed by a few recent advances and some thoughts on future research.

Review of Methods

Methods to represent uncertain variables as real numbers, and then perform an aggregation (as a sum, product, integral, *min*, *etc*.), for decision-making purposes are not new. Uncertainty may be: uncontrolled stochastic variations in variable values, design imprecision as described above, variable values to be chosen by optimization, *etc*. Probability and Bayesian inferencing [37, 95, 104], Dempster-Shafer theory [83, 87], fuzzy sets and triangular norms in general [20, 42, 44, 54, 56, 113], and finally utility theory [24, 26, 43] are among the existing formal[3] methods for representing uncertainty.

These methods all represent uncertainty with a range for each variable and a function defined on that range. An illustration is shown in Figure 1, where d is an uncertain variable and μ_d is the uncertainty on variable d. They are also similar in that they all conform to the first three restrictions of Table 1. The axioms shown in Table 1 have been proposed as the minimum set of restrictions for an engineering design combination (aggregation) function $\mathcal{P}$ [58], where μ_i is the uncertainty associated with the i^{th} aspect of the design. The discussion below indicates where these theories diverge among themselves and with optimization theory, matrix methods, and the MoI.

Imprecision vs. Uncertainty. Uncertainty, which usually represents uncontrolled stochastic variations with the mathematics of probability, is distinct from imprecision. Uncertainty occurs throughout engineering design, in the form of manufacturing variations, material property variations, *etc*. Including uncertainty in engineering design decision-making can help produce robust designs by assessing the expected size of variations and determining the risk of failure. Many design methods have been developed specifically to address these calculations, including Taguchi's method, probabilistic optimization, and utility theory.

[1] Variational and parametric modeling systems begin to approach this, by letting the designer specify a dimension precisely, but with the idea that it will be modified later.

[2] Joseph G. Gavin, Jr., discussing the design of the lunar module that landed NASA astronauts on the moon on July 20, 1969.

[3] Here the term "formal" is used to mean computable, in the sense that a design method could be automated.

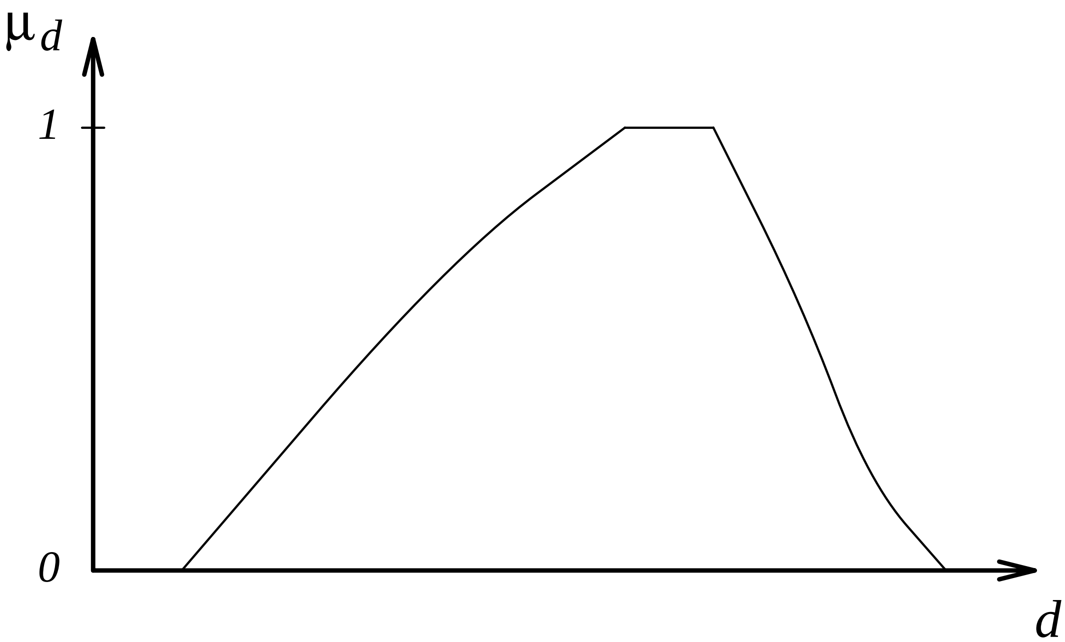

Figure 1: Example imprecise design variable

$\mathcal{P}(0,\ldots,0)=0 \qquad \mathcal{P}(1,\ldots,1)=1$	(boundary conditions)
$\forall\, k, \mathcal{P}(\mu_1,\ldots,\mu_k,\ldots,\mu_N) \leq \mathcal{P}(\mu_1,\ldots,\mu'_k,\ldots,\mu_N)$ iff $\mu_k \leq \mu'_k$	(monotonicity)
$\forall\, k, \mathcal{P}(\mu_1,\ldots,\mu_k,\ldots,\mu_N) = \lim_{\mu'_k \rightarrow \mu_k} \mathcal{P}(\mu_1,\ldots,\mu'_k,\ldots,\mu_N)$	(continuity)
$\mathcal{P}(\mu_1,\ldots,0,\ldots,\mu_N)=0$	(annihilation)
$\mathcal{P}(\mu,...,\mu)=\mu$	(idempotency)

Table 1: Overall preference resolution axioms.

In the context of engineering design, the term imprecision is used to mean uncertainty in choosing among alternatives. An imprecise variable in preliminary design is a variable that may potentially assume any value within a possible range because the designer does not know, *a priori*, the final value that will emerge from the design process. The nominal value of a length dimension is an example of an imprecise variable. Even though the designer is uncertain about what length to specify, she usually has a preference for certain values over others. This preference, which may arise objectively (*e.g.*, cost or availability of components or materials) or subjectively (*e.g.*, from experience), is used to quantify the imprecision with which design variables are known. Once one or more alternative design concepts are available, each can be described by a collection of (imprecise) variables.

To represent an imprecise variable, d, a range of real numbers could be used, in the style of interval analysis [101, 102, 100]. Alternatively, imprecision could be represented by a range as well as a function, μ_d, defined on this range to describe the desirability of or preference for particular values, as illustrated in Figure 1. In this way variables whose values are not known precisely can be formally represented.

Combination Functions. Nearly all formal design methods for representing uncertainty or imprecision utilize one or more functions to aggregate information from multiple attributes. The combination calculation performs a trade-off, such that some aspects of a design may contribute more heavily to the combined result than others. Combination functions are also referred to as *metrics*.

A combination function $\mathcal{P}$ is a formalization of the process of trading-off competing design attributes, and should satisfy the restrictions for engineering design proposed in [58] and shown in Table 1. Combination functions can be divided into two classes: compensating and non-compensating. A compensating combination function (*e.g.*, sum) will produce an overall measure of a design alternative where aspects that perform well can compensate for aspects that perform poorly. For example, a potential customer of a new car may prefer plenty of legroom and good fuel economy, and be willing to let a bit more legroom partially compensate for poor fuel economy when creating an aggregate evaluation of a particular new car. A non-compensating combination function (*e.g.*, *min*) will produce an overall measure of a design alternative that is limited by the most poorly performing aspect.

Formalizing the combination of attributes permits trade-off strategies that are determined informally or implicitly to be decided rationally and explicitly. A formal trade-off method also permits design decisions to be clearly understood and recorded for later retrieval and examination. When a question regarding a particular design trade-off arises at a later stage in the design process, a formal method can provide a clear and complete picture of how the decision was reached. Moreover, the trade-off can be repeated with revised information, thus confirming or refuting the original decision.

Methods for representing and manipulating uncertainty and imprecision, and for combining multiple attributes of an engineering design, are reviewed below.

Utility Theory. Utility theory [24, 26, 43] was developed to assist in selecting among a choice of distinct actions, given uncertainty (noise) in the outcome of each action. In utility theory, each aspect of a decision (*e.g.*, each design variable) is assigned a function representing utility, as illustrated in Figure 1. The utilities for the individual aspects are aggregated to determine the overall utility, and the combination of variable values that maximizes the overall utility is used.

Utility theory is restricted to decision problems in which the individual preferences can be modeled as additive, either with a weighted sum, or with a "multiplicative" form which is also additive but includes nonlinear terms [26]. Since the overall metric is additive, utility theory always reflects a compensating strategy, allowing the higher preference of some goals to offset the lower preference of others.

Because utility theory is additive, it fails the annihilation restriction shown in Table 1. This means that the utility of one aspect of an alternative can be zero, but the overall utility of the alternative will be non-zero. To surmount this difficulty, Thurston has applied utility theory to design by dividing the complete set of goals into two classes: objective constraints and subjective goals [94, 92, 93]. Objective constraints have crisp achievement levels that must be satisfied, and thus become standard constraints as in non-linear programming formulations. Subjective goals are those which can be traded-off, and are modeled using standard utility theory. The subjective goals are traded-off among themselves in a compensating manner (using either of the formulations of utility theory), and the objective constraints are traded-off informally (by iteratively refining the constraint values). The overall strategy, therefore, remains informal.

Utility theory was originally developed for management decisions, not for engineering design, and requires that all attributes be aggregated into a single goal (utility). Economists generally believe every aspect of a decision can always be translated into a monetary cost. Monetary costs are additive. Aspects which are not additive, or which cannot be "bought off", are not deemed possible. The axiom of utility theory which creates the demand that a gain in any aspect must be able to compensate for any loss in any other aspect is the Archimedian property. This restriction requires that any decrease in overall preference caused by changes in the performance of one variable must *always* be able to be balanced by an increase in performance in any of the other variables. Clearly this is not the case in engineering design, as others have argued [11, 97]. For example, given a fixed material, the tensile strength limits cannot be exceeded no matter the reduction in the design's cost. Material stress simply cannot always be traded-off in a compensating fashion. The Archimedian property and annihilation cannot be simultaneously satisfied. This implies that utility theory will not permit a worst case analysis, which is required in many instances in engineering design [33, 49].

Fuzzy Sets. Fuzzy sets have been used to represent imprecision in (non-design) decision-making [20, 42, 44, 54, 56, 113]. Fuzzy sets are intended to model subjective uncertainty for use in logic, constructing subjectively uncertain versions of "and" and "or" of classical logic. In the first paper describing the use of fuzzy sets for decision-making [9], a decision was defined as a convolution of the constraints and goals, using *min* as a non-compensating metric. They also suggested that at other times simple multiplication of the individual preferences might be appropriate.

The overall metrics of *min* and multiplication have been expanded to the more general class of *t-norms*, first proposed by Menger [51], and reviewed in Dubois and Prade [21]. *T-norms* are bounded above by *min*, and are the uncertain version of conjunction. *T-norms* are not appropriate as an overall design metric because they do not, in general, satisfy the restrictions of Table 1. Probabilistic reasoning, Dempster-Shafer theory, and fuzzy sets all employ *t-norms* [21]. Related to each *t-norm* is an associated *t-conorm* (or *s-norm*), which is the uncertain version of disjunction. *T-conorms* are bounded below by *max*. The set of functions between *t-norms* and *t-conorms* are the *mixed connectives*, bounded between *max* and *min*, and are the class of combination functions used by the MoI and utility theory.

Using fuzzy sets for decision-making has received further attention. [22, 38, 109, 110, 111, 112], for example, all discuss converting linguistic expressions into fuzzy sets, and then using the fuzzy mathematics to make decisions. An excellent review of fuzzy multiple attribute decision-making is presented in [14]. Bellman and Zadeh suggested using a weighted sum of the preferences. [39] and [7] also develop additive metrics using fuzzy sets beyond Bellman and Zadeh's original work. [110] observed the "softness" of multiplication as a connective, and proposed it (and *min*) in conjunction with weights.

Completely independent of fuzzy set formulations, [32] (the same year as Zadeh's initial paper on fuzzy sets) proposed a product of powers as a "desirability function" in chemical process problems. He observed that the annihilation condition is required for engineering design.

[15, 16] and [69, 70, 71, 72, 73] have applied fuzzy set formulations to design optimization problems in mechanical engineering. [55] has applied fuzzy sets to nuclear radiation cover design. All have used only a non-compensating trade-off strategy for making deci-

sions. Other applications of fuzzy optimization are reviewed by [50].

Optimization. Other design methodologies exist that do not explicitly represent uncertainty or preferences on variables. For example, optimization formulations (linear, nonlinear, integer, and mixed integer programming) [1, 4, 65, 66, 74] assume a relationship between preference and the objective function: the lower the function, the higher the preference. Also, a relation is assumed between preference and the constraint functions: if a constraint is satisfied, the preference is high. If any constraint is slightly violated, that constraint alone dictates the preference for the design is zero. Single objective optimization utilizes a non-compensating strategy: at any point in the space of design variables, one goal determines the preference (either the objective function, or a constraint).

Instead of a single objective formulation with constraints, others have proposed multi-objective optimization. Here, strategies are formally explicit only when a norm across the goals is used [79]. For example, weighted sum techniques [17, 23, 25, 57, 84, 88, 108] are compensating formulations: the higher performing objectives are averaged with the lower performing objectives, with the incorporation of importance weighting coefficients. As a specific example, the Archimedian "goal programming" [57, 80] formulation is a weighted sum technique, with target values and nonlinear weights. Additive metrics have been discussed above in relation to utility theory. Other formulations can be found in [82, 84].

[11] and [97], argue that the formulations that fail the annihilation condition (*e.g.*, addition) are not well suited for engineering design.

Typically, multi-objective formulations are used iteratively, without specifying a formal strategy. Such methods have been used in design [5, 68]. There are also algorithms for such iteration. STEM [10], GDF [28], and the VI algorithm [45], for example, interactively question a decision maker about relative trade-off preferences. Such algorithms are based on an informal overall metric consistent with utility theory [88], and thus exhibit an informal compensating design strategy.

Matrix Methods. Concept selection charts [2, 3, 6, 11, 67] are commonly used in engineering design decision-making. When using a formal chart, alternatives are listed versus evaluation criteria. Each alternative is ranked on each criterion, and the alternative with the best aggregated (weighted sum) score is selected. [67] presents an alternative technique of summing the negative and positive aspects of each alternative, and then making an informal decision based on these ranks. In an analysis of these methods, [52] considers a choice among four alternatives, and demonstrates that four different metrics selected each of the four different alternatives. The choice of decision-making method can entirely change the outcome, which confirms the importance of utilizing a method appropriate for engineering design.

The Analytic Hierarchy Process, or AHP, originally developed by Saaty [29, 76, 77], is a formal method for determining relationships between discrete alternatives, each of which can be rated by one or more attributes. A tutorial example is presented in [31]. AHP has an axiomatic foundation [78]. Similar to utility theory, this foundation requires that the Archimedian property must be satisfied, which dictates that the annihilation condition is not satisfied. Since the AHP's ratings are derived from a linear weighted sum, it only implements a compensating trade-off.

AHP can only consider discrete alternatives; no continuous variations can be incorporated. Additionally, all alternatives are compared to the lowest performing alternative; AHP includes no ability to indicate that one (or more) alternatives are completely unacceptable, or have violated one or more constraint(s). However, AHP is one of the few methodologies that can incorporate hierarchical objective criteria.

Finally, "Quality Function Deployment," or QFD [2, 34] begins by listing the customer requirements for a design on one axis of a chart and the performance metrics for the design on the other axis. Ratings are performed by using a transformation to convert symbols to numerical equivalents, and summing. This summation has the difficulties of the additive metrics discussed above.

Probability Methods. In Taguchi's method [13, 36, 41, 75, 86, 90, 91], the "quality" of a design variable set $\vec{d}$ is defined by the expected variation of a single performance variable p from a target value τ due to uncontrolled stochastic noise. Thus, Taguchi's method finds the mean of a single performance variable (variation of p from τ), not preference over many design variables and performance variables [59].

Experimental design techniques can also be used to determine experimental points in the noise space. See [8, 12, 40] for a discussion of factorial methods for determining experimental points. Alternatively, Monte Carlo simulation [30] could be used for more accuracy.

Probabilistic optimization methods can be used to evaluate a solution when noise is present, by evaluating the expected value of the objective function, as presented by [85, Chapter 13]. The discussion on optimization methods above applies to probabilistic optimization methods as well.

Necessity Methods. Necessity determines the probability (α) of a design operating successfully despite uncontrolled variations in one or more parameters over a range (confidence interval). This approach can account for the worst case of random disturbances. [98] and [100] have developed a "Labeled Interval Calculus" that consists of interval mathematics with associated "only" or "every" labels. [99] have shown that a set-based approach to design decision-making (using intervals) can facilitate concurrent engineering.

Noise can be rated by the expected value, or by the worst case scenario (using necessity). [85, Appendix C] and [81] reject using confidence intervals. Siddall argues that one has difficulty determining the density function pdf and the confidence level α. Savage argues against the use of confidence information since he feels one is still obliged to choose the expected value. However, confidence intervals are widely used in practice [33, 49], as is modeling to consider the worst case noise [19].

Fuzzy Design Methods. Fuzzy design methods combine many of the valuable attributes of the methods described above. They are formal (computable) methods for representing and manipulating design imprecision (uncertainty in choosing among alternatives) using the mathematics of fuzzy sets. Several groups are applying fuzzy methods to engineering design problems [105, 58, 53, 114, 115].

Imprecision is represented by a range, and a function defined on this range (μ_d), to describe the desirability of or preference for particular values, and to incorporate the designer's experience and judgement into the design evaluation. Non-parametric attributes (such as material choice, color, style, *etc.*) as well as real-valued attributes (such as physical dimensions, material properties, cost, *etc.*) can be used.

In the Method of Imprecision (MoI) [105, 106, 63], constraints can be similarly imprecise, permitting the customer to specify preferences over a range of values, rather than a crisp constraint that may be moved by negotiation later in the design process. Because the method was developed specifically for engineering design, the trade-off combination functions meet the restrictions shown in Table 1 [58]. A choice of two basic combination functions is available to aggregate the preferences for the attributes of the design: the (non-compensating) *min* and a (compensating) product of powers.

Because the MoI does not require all attributes to be aggregated into one evaluation metric, evaluations of the various aspects of a design can be made in a hierarchy. For example, safety margin might be traded-off in a non-compensating way among several parts of the design that are subject to loading, and weight and cost might be traded-off in a compensating way. The results of those two trade-offs then might be traded-off with a non-compensating combination function. Because importance weighting can be readily applied, the relative weight of each aspect of the decision can be incorporated into the hierarchy [48].

Finally, stochastic uncertainty (such as uncontrolled manufacturing variations) and possibilistic uncertainty and necessity (such as post-manufacturing tuning adjustments) can be incorporated into the design decision-making by utilizing well known expectation calculations [62].

Utility theory and the MoI are strongly similar when there is only one goal, and a compensating strategy is used [60]. When goals are traded-off in a non-compensating manner, and without considering importance weightings, the MoI reduces to a convolution of the constraints and goals used in fuzzy sets for decision-making [9].

The Method of Imprecision

The following sections will present a brief tutorial on how imprecision is used to facilitate decision-making in engineering design using the MoI.

Definitions and Notation. *Design variables* are denoted d_i, and the valid design variable values within the *design variable space* (DVS) form a subset $\mathcal{X}$. The set of valid values for d_i is denoted $\mathcal{X}_i$. The preference that a designer has for values of d_i, the ith design variable, is represented by a preference function on $\mathcal{X}_i$, called the *design preference*: $\mu_{d_i}(d_i)$.

Performance variables are denoted p_j. For each performance variable p_j there must be a mapping f_j such that $p_j = f_j(\vec{d})$. The mappings f_j can be any calculation or procedure to measure the performance of a design, including closed-form equations (*e.g.*, for stress, weight, speed, cost, *etc*.), iterative solutions, heuristic methods, “black box” calculations, testing of prototypes, or consumer evaluations. The subset of valid performance variable values $\mathcal{Y}$ is mapped from $\mathcal{X}$ and the set of valid values for p_j is denoted $\mathcal{Y}_j$. The *performance variable space* (PVS) is the dependent set of performances evaluated for each design in the DVS. In order to compare design alternatives, design preferences are mapped onto the PVS via the extension principle [111], discussed below.

Specifications and requirements also embody design imprecision, even though most are written as if they were crisp, *e.g.*, “This device must have a range of at least 250 km.” Such a requirement implies that given two designs arbitrarily close together, one with a range of 250 km and one just below, the first would be acceptable but not the second, as shown by the dashed line in Figure 2. Specifications and requirements in the real world are commonly fuzzy. Often the designer must ask questions to distinguish the underlying

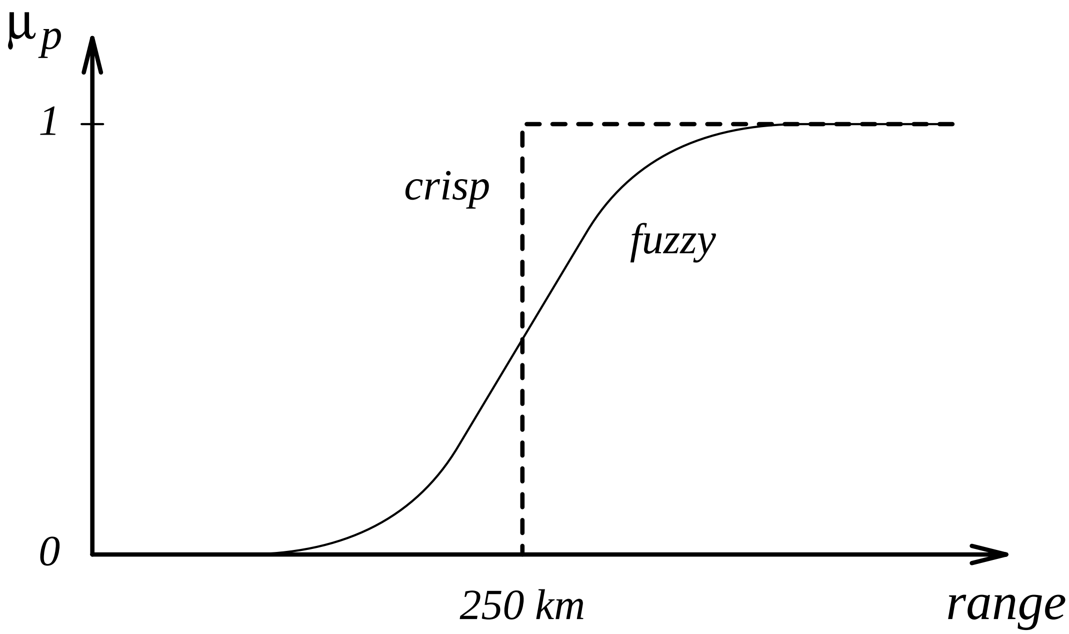

Figure 2: Example imprecise functional requirement

fuzzy constraint so that the final design will satisfy the customer's actual requirements even though it may violate the crisp constraint initially given. The fuzziness of constraints and the fuzziness of preliminary design variables are both forms of design imprecision and can be represented in exactly the same way. The customer's preference (requirements) for values of p_j, the jth performance variable, is represented by a preference function called the *functional requirement*: $\mu_{p_j}(p_j)$. The solid line in Figure 2 shows the fuzzy functional requirement.

Trade-Off Strategies. The combined preference of the designer and customer for a particular design $\vec{d}$ is represented by an overall preference $\mu_o(\vec{d})$, which is a function of the design preferences $\mu_{d_i}(d_i)$, and the functional requirements $\mu_{p_j}(p_j)$:

$$\mu_o(\vec{d}) = \mathcal{P}\left[\mu_{d_1}(d_1), \ldots, \mu_{d_n}(d_n), \mu_{p1}(f_1(\vec{d})), \ldots, \mu_{p_q}(f_q(\vec{d}))\right]. \tag{1}$$

Two combination functions ($\mathcal{P}$) that satisfy the restrictions of Table 1 have been identified: *min* and a product of powers. Figure 3 shows the overall preference μ_o obtained by combining a design preference μ_d and a functional requirement μ_p. Both a compensating trade-off ($\mu_o = \sqrt{\mu_d\ \mu_p}$) and a non-compensating trade-off ($\mu_o = \min[\mu_d, \mu_p]$) are shown. Note that the compensating trade-off results in an overall preference μ_o that is greater than or equal to μ_o for the non-compensating trade-off. Where unequal preferences are traded-off, higher preferences compensate for lower preferences, raising the combined preference above the minimum. Importance weightings can further shift combined preferences, as discussed in [58].

Quantifying Imprecision. Utility and risk-aversion are quantified in utility theory via the lottery method [43]. Unfortunately no such formal method exists for eliciting preference. However, limits of acceptability for variable values, whether communicated formally

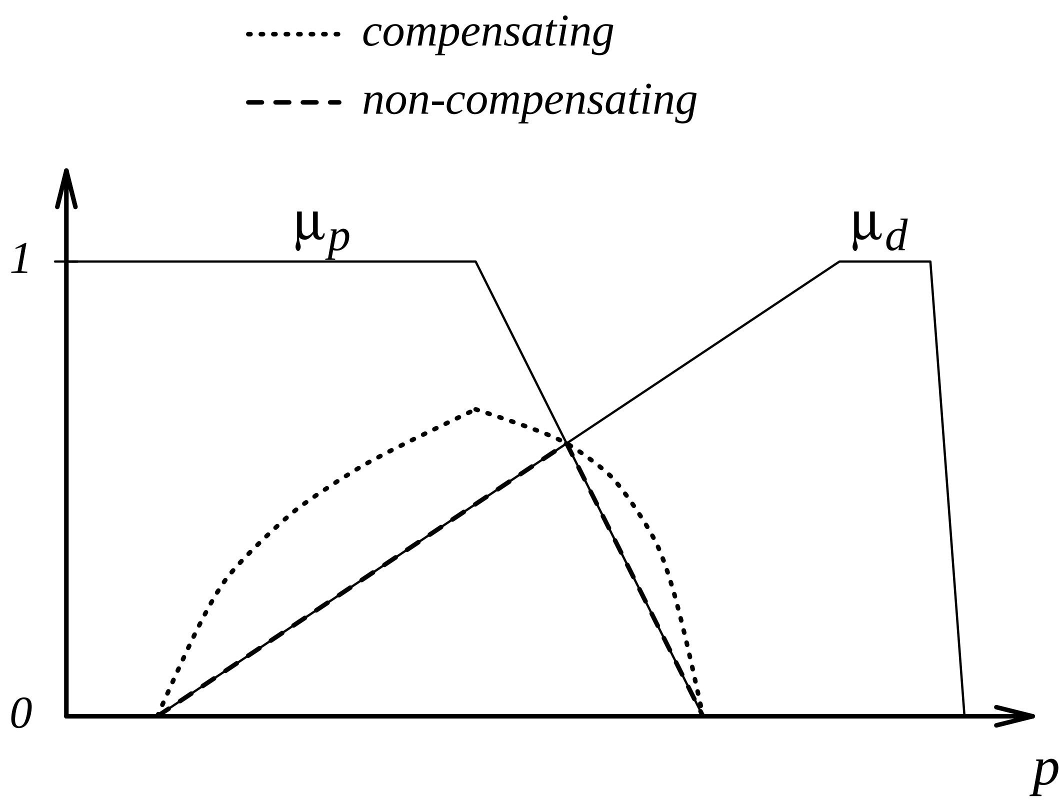

Figure 3: $\mu_o = \mathcal{P}(\mu_d, \mu_p)$ for compensating and non-compensating trade-offs

or established informally by experience, are familiar to engineers in industry [99]. Such acceptable intervals correspond to intervals over which preference is greater than zero. This suggests that rather than determine the preference μ_d at each value of d, as shown in Figure 1, it may be more natural to determine the intervals in d, called α-cuts, over which μ_d equals or exceeds certain preference values α.

The use of intervals encourages the passing of set-based design information between engineering groups early in the design process [99], and permits the early release of possible sets of design data from one engineering group to the next in advance of precise design information. This approach has many advantages over the traditional "point-by-point" design iteration. The MoI can extend set-based concurrent design by providing preference information over the possible range of design data.

Imprecision Calculations. After specifying design preferences μ_{d_i} on $\mathcal{X}_i$ and functional requirements μ_{p_j} on $\mathcal{Y}_j$, and identifying a design trade-off strategy, the next step is to determine the induced values of μ_{d_i} on $\mathcal{Y}$ (design preferences mapped onto the performances), given by the extension principle [111]:

$$\mu_d(\vec{p}) = \sup_{\vec{d}:\vec{p}=\vec{f}(\vec{d})} \left[\mu_d(\vec{d})\right] \tag{2}$$

A simple one-dimensional example of Zadeh's extension principle is shown in Figure 4. The performance p achieved for each value of the design variable d is given by the function f, which is a curve in this simple example.[4] The corresponding $\mu_d(d)$ can be mapped onto p, producing $\mu_d(p)$: the design preference mapped onto the performance space (as illustrated by the dashed lines in Figure 4). For more realistic design problems, each p will be a function of many d's, and each function f will be a hyper-surface.

An algorithm to compute Zadeh's extension principle (and thus to calculate $\mu_d(\vec{p})$) is the *Level Interval Algorithm* (LIA), first proposed by [18] as the "Fuzzy Weighted Average" algorithm and also called the "Vertex Method", and extended by [107, 64].

Once the imprecision on each design variable ($\mu_d(\vec{d})$) is induced onto the PVS, the induced preferences are combined with the functional requirements ($\mu_p(\vec{p})$) to obtain an overall preference ($\mu_o(\vec{p})$). The point (or points) with the highest preference correspond to the performance of the overall most preferred design(s). The design problem is to find the corresponding set of design variables ($\mu_d(\vec{d^*})$) that produce the maximum overall preference (μ_o^*). In the typical engineering design case, where the inverse mapping ($\vec{f}^{-1} : \mathcal{Y} \rightarrow \mathcal{X}$) doesn't exist, $\mu_o(\vec{d})$ can still be obtained point by point [46].

Example

An industrial example of cost estimation for aircraft engine design, utilizing the Engine Development Cost Estimator developed by General Electric Aircraft Engines [47, 46], shows how a crisp design cost estimator can be integrated with the MoI. This permits imprecise cost estimates to be developed when only imprecise design data is available. The original example included a formal, and imprecise, functional requirement (μ_p). The example has recently been extended to include more informal functional requirements.

Figure 5 shows the results of the MoI applied to the engine development cost estimator, using a compensating strategy. Two different design options were simultaneously explored,

[4] Note that here f is non-linear. Non-monotonic and discrete functions can also be used [107, 64].

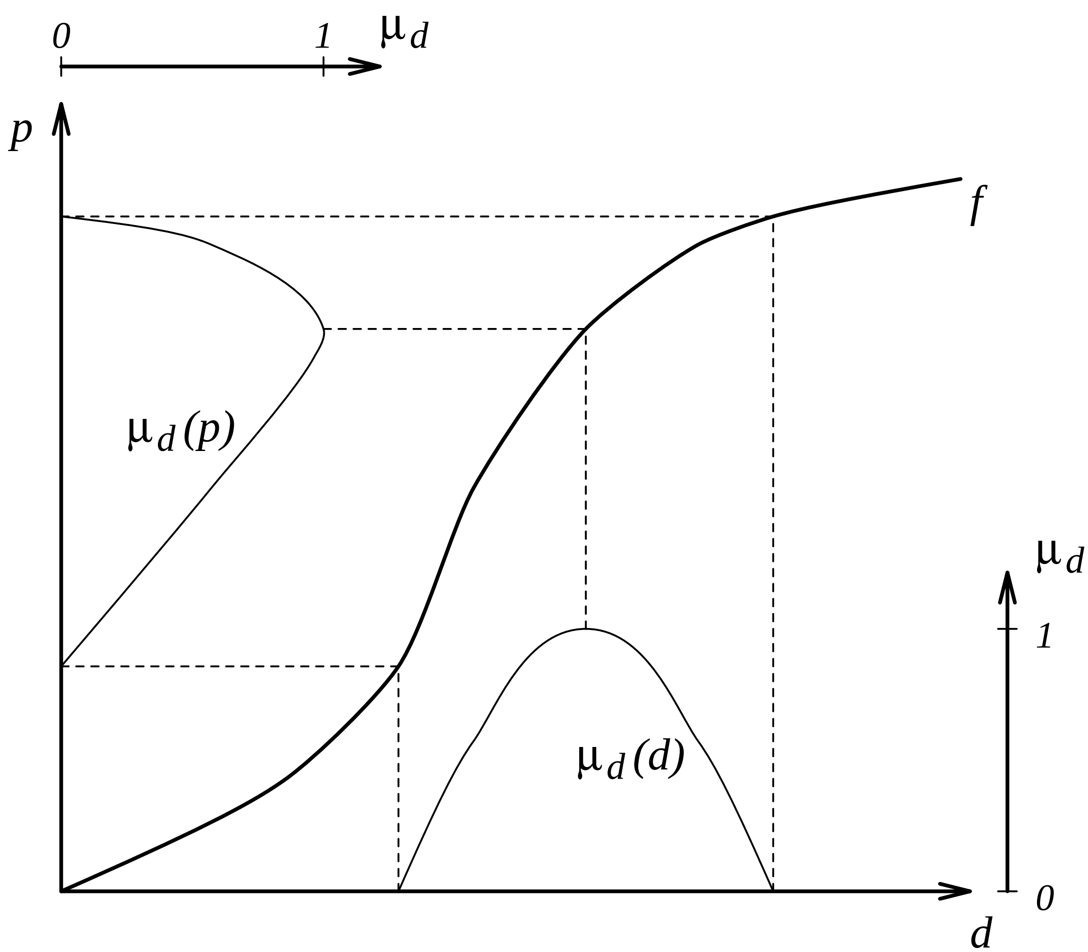

Figure 4: Zadeh's extension principle

and are both shown on the figure. Option 1 is to develop the new engine from an existing turbojet design by the addition of a front fan with matching shaft and low pressure turbine. Option 2 is to modify an existing, but dated, turbofan design. The two curves are the designers' preferences (for a range of each design variable) induced onto the performance space (here: cost). Each point on the curves in Figure 5 corresponds to (at least) one design.

In comparing the cost specifications to the calculated imprecise design costs, if the customer has imposed a strict limit on the development cost, it may be applied by choosing the point with highest preference below this limit. If a formal, and imprecise, functional requirement for cost is available, the customer's preference and the designer's preference (induced onto the cost variable) can be traded-off, as described above. However, if the customer can only provide "cheaper is better" as an informal cost specification, an informal trade-off can still be made. Let $\vec{d}_{ref}$ be the design with the least cost among the highest preference designs (in Figure 5, $\vec{d}_{ref}$ is the left-most point at $\mu_d = 1.0$ for each option). A design $\vec{d}$ with lower cost is compared to $\vec{d}_{ref}$ by examining the slope of the line through both designs: the larger the slope, the greater the trade-off. The customer or designer should choose a design on a line with a slope that represents an acceptable trade-off.

Future Work

Promising areas for future work in further developing methods for representing and manipulating imprecision in engineering design include: developing a consistent (and formal) method for eliciting preference from designers and customers, and refinement of the weighting methods. Noise (meaning probabilistic uncertainty) has been thoroughly included in the MoI [62, 61], but an application to a commercially relevant industrial problem has not yet been made. The MoI was developed to permit hierarchical design decision-making, but a scheme to represent the hierarchy is lacking. Incorporating preference to create fuzzy-set based concurrent design methods appears to have significant promise, and is presently an active area of research. One area of future work that seems especially interesting is the notion of a fuzzy solid modeling system. Imprecise physical dimensions can be represented mathematically, but a useful display of this information has not yet been developed.

Conclusion

Imprecision and uncertainty occur throughout the engineering design process. Many methods for incorporating uncertainty (*e.g.*, utility theory, probability methods, Taguchi's method, *etc.*) are in common use and are reviewed here, but methods to represent imprecision in engineering design are few. The Method of Imprecision (MoI) is a formal method for incorporating the natural level of imprecision that occurs throughout the engineering design process, and can include: many incommensurate aspects of a design, imprecise constraints, compensating and non-compensating trade-offs, hierarchical trade-offs, and importance weightings. Uncontrolled variations (noise) can also be incorporated so that the design with the greatest overall preference and most robustness to the noise can be found.

Furthermore, by encouraging the designer and customer to specify preferences on design and performance variables, design communication will evolve from individual "point" designs to (fuzzy) sets of designs. Since a range of possible design variable values can be

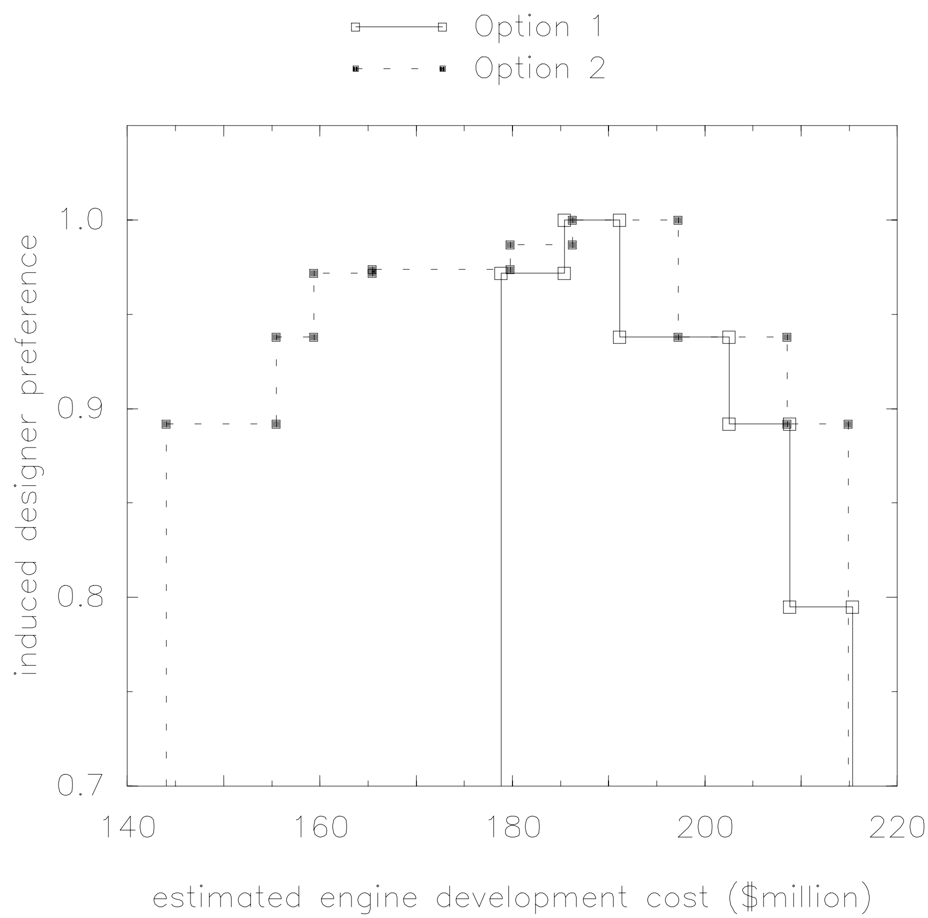

Figure 5: $\mu_d(p)$ for a compensating strategy.

released to down-stream design processes earlier than a completed individual design, the MoI can facilitate (fuzzy) set-based concurrent design.

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