A New Threat Assessment Measure for Collision Avoidance Systems

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Abstract— **Collision avoidance systems (CAS) are an emerging automotive safety technology that assists drivers in avoiding collisions. They monitor the dynamic traffic and vehicle information in real time and assess the current threat level to decide if warnings should be issued to the driver or automatic braking is needed to avoid collisions. Several measures have previously been defined for threat assessment with various warning and overriding algorithms proposed.**

A new threat assessment measure, time-to-last-second-braking (T_{lsb}) is proposed in this paper and its advantages over previous measures are discussed. It directly quantifies the danger or threat level of the current dynamic situation objectively as well as assesses the urgency level for the required evasive action, *e.g.*, braking. It is in agreement with human natural judgment of the urgency and severity of threats. Furthermore, new warning and overriding criteria are proposed based on the new T_{lsb} measure, providing more appropriate warning and more effective override timing.

I. INTRODUCTION

Collision avoidance systems (CAS) are an emerging automotive safety technology that assists drivers in avoiding potential collisions. The information sources of the CAS come from multiple on-board sensors. The range, range rate, and angular information of other vehicles and/or objects around the host vehicle can be measured by radar, lidar, and/or cameras in real time. Other typical on-board sensors measure host vehicle speed, acceleration, steering angle, yaw rate, *etc*. Collision avoidance systems process all the information in real time to keep track of the most current vehicle-to-vehicle kinematic conditions. When a potential collision threat is identified by the system, appropriate warnings are issued to the driver to facilitate collision avoidance. If the driver fails to react in time to the warnings to avoid the imminent collision, an overriding system can take over control to avoid or mitigate the collision in an emergency situation. Therefore collision avoidance systems can assist drivers in two ways, warning and/or overriding, according to the dynamic situation.

In developing a collision warning system (CWS), two important parameters involving driver behavior must be considered. One parameter is the time it takes for the driver to respond to the crash alert and begin braking, *i.e.*, driver reaction time (t_r), and the second parameter is the driver deceleration (or braking) behavior in response to the alert across a wide variety of initial vehicle-to-vehicle kinematic conditions. An overriding system has the advantage of being less sensitive to human factors, hence it is more promising in terms of achieving better and robust system performance. However, it is also the most intrusive way to assist the driver, so its timing has to be carefully designed to get driver acceptance, and to obtain regulatory approval.

In addition, both warning and overriding systems are subject to some objective hardware limits and environmental factors, such as the maximum traction available from the ground-tire contact and brake efficiency, *etc*. A traction sensor could be used to obtain an estimate of the current road traction conditions, as reviewed in [1].

II. PRIOR WORK

A considerable amount of research has been performed on collision waning systems, which are the first step to assist drivers in collision avoidance. The key is to ensure that warnings are issued to drivers at the *appropriate* time, *i.e.*, just in time for drivers to react and avoid collisions while not too early or too frequent to become a nuisance or distraction to drivers. In the prior work, several measures were defined to characterize the emergency level of various dynamic situations, and different sets of human-vehicle experiments were carried out to calibrate these measures to human performances and reactions, based on which different warning criteria were developed to assist drivers.

A. Measures Defined

Quantitative measures have been defined to characterize the emergency level of various dynamic situations. The measures defined in the literature include time-based, distance-based and deceleration-based measures.

One frequently used time-based measure is the *time-to-collision* (*TTC*), which is the time it would take for a collision to occur at the prevailing speeds, distances, and trajectories associated with the host vehicle and the closest lead vehicle [2]. More specifically, three different *TTC* measures have been used [3]. The *TTC*1 measure was defined as the range R (*i.e.*, the bumper to bumper distance between the two vehicles) divided by the closing speed between these two vehicles, or $-R/RR$, where RR is the range rate or closing speed. The *inverse TTC*1 measure was defined as the inverse of *TTC*1, or $-RR/R$. The *TTC*2 measure was defined as the time it would take the host and lead vehicles to collide assuming the prevailing vehicle speeds and acceleration/deceleration values (*i.e.*, at the current "constant" rate of speeding/slowing). Hence the difference between *TTC*1 and *TTC*2 is that the latter takes into account the

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TABLE I
THREAT ASSESSMENT MEASURES DEFINED IN THE LITERATURE.

Type	Definition
*TTC*1	projected time to collision assuming prevailing speeds and distance ($-R/RR$)
$1/$*TTC*1	inverse of *TTC*1 ($-RR/R$)
*TTC*2	projected time to collision assuming prevailing speeds, accelerations, and distance
t_h	time headway between the host and lead vehicles (R/v_H)
a_{req}	constant deceleration level required to avoid a rear-end collision
D_{min}	projected minimum distance during collision avoidance process

acceleration information of both vehicles while the former always assumes zero acceleration for them.

A second related time-based measure is the *time headway* (t_h), which is calculated as the range between the two vehicles divided by the following host vehicle speed, or R/v_H [4]. Time headway is important because it specifies how much time the following driver has to react in case the lead vehicle suddenly brakes at maximum deceleration level.

One important deceleration-based measure is the *required deceleration* (a_{req}) measure, which is defined as the constant deceleration level required for the host vehicle to avoid a potential collision with the vehicle ahead [5]. This measure is calculated under the same assumptions as the *TTC*2 measure.

A distance-based measure is the *projected minimum distance* (D_{min}) between the host and lead vehicles during the approaching/avoiding process [6]. It is calculated using the prevailing range and vehicle speeds, and the assumption that the lead vehicle will maintain the current acceleration level until it comes to a stop, while the host vehicle starts to brake at a constant maximum deceleration level after an assumed driver reaction time, during which it keeps its original acceleration level. Another related measure is the *projected time to* D_{min}, which indicates the urgency of the situation [7].

Table I summarizes the various threat assessment measures mentioned above. Each measure characterizes the current dynamic situation in one way and can be used as basis for threat assessment and warning/overriding criteria. However, there does not exist a clear quantitative relationship between these measures and the threat level, nor do they provide the best timing for warning and overriding actions.

B. Driver Reaction Time

Driver reaction time is an important parameter and plays a major role in the success of the collision warning systems. In this paper the driver reaction time includes the human mental processing time in response to a signal or stimulus, the movement time for the driver's foot to move to the brake pedal, and the brake system delay.

Many research experiments have been performed to measure human driver reaction times to different stimuli under various situations. Comprehensive reviews of driver reaction times can be found in the literature [8], [9], [10]. It was noticed that driver reaction times can be approximated with a lognormal probability distribution with parameters μ and σ^2 [6], [11].

TABLE II
ESTIMATES OF UNEXPECTED DRIVER REACTION TIME IN SECONDS.

Alert Type	mean	std	μ	σ	75%	85%	90%
none	1.25	0.6	1.13	0.46	1.53	1.81	2.02
Visual	1.13	0.52	1.03	0.44	1.38	1.62	1.80
Auditory	0.99	0.44	0.90	0.43	1.20	1.40	1.55
Vis.+Aud.	0.90	0.34	0.84	0.37	1.08	1.23	1.35

Two different situations are especially interesting for collision warning systems. One is normal driver reaction times toward unexpected natural driving scenarios, such as the onset of the brake lights of the lead vehicle or yellow traffic lights. The other is the driver reaction times in response to unexpected artificial signals, such as a red icon appearing in front of the driver or specific auditory signals, which could be considered as potential warning signals.

From the results reported in literature, the best estimate for natural driver brake reaction time to common but uncertain signals (*e.g.*, lead vehicle brake lights or yellow traffic lights) lies between 1.14 and 1.38 seconds, with standard deviation 0.6 s [12], [13]. The lognormal distribution model with parameters $\mu = 1.13$ s and $\sigma = 0.46$ s approximately represents the natural human driver reaction time with mean 1.25 s and standard deviation 0.6 s.

The experiments on driver reaction times in response to the sudden appearance of a red square reported approximately 1.13 s on average [14]. The driver brake reaction time in response to completely unexpected auditory signals was estimated to be 0.9 s or longer in 50% of all sudden accident situations, and about 1.2 s for the 75^{th} percentile [15]. Finally, driver reaction times under different types of dual-modality (*i.e.*, both visual and auditory) crash alerts were extensively investigated in [5], where the shortest reaction times with the least variance were recorded under surprise, unexpected conditions. It was further verified that brake reaction times were faster with frontal collision warning alerts [16].

Table II provides a summary of driver reaction times in response to different types of unexpected stimuli, characterized by the lognormal probability density model with parameters μ and σ. The mean and standard deviation (std) values as well as the 75^{th}, 85^{th}, and 90^{th} percentile values are listed in the table.

C. Warning and Overriding Algorithms

Various warning and overriding algorithms have been developed and investigated in the literature [17]. Most algorithms compute a warning range (R_w) based on the current kinematic situation, and a warning is issued if the current range R is less than R_w. Some algorithms also calculate an overriding range (R_o), and automatic braking (overriding) is applied if R is within R_o.

1) Mazda Algorithm: The Mazda overriding algorithm [18] considers a hypothetical worst case. It assumes that initially both the host vehicle and the lead vehicle

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maintain constant speeds v_H and v_L, respectively. Then the lead vehicle starts to brake after time τ_2 at deceleration level $-\alpha_2$, while the host vehicle starts to brake after an additional time τ_1 at deceleration level $-\alpha_1$, which continues until both vehicles come to a full stop. The overriding range R_o is computed as the minimum range needed at time 0 to allow the above scenario to happen without collisions.

$$R_o = v_H\tau_1 - RR\cdot\tau_2 + \frac{v_H^2}{2\alpha_1} - \frac{v_L^2}{2\alpha_2} + R_{min} \qquad (1)$$

where RR is the range rate ($RR \equiv v_L - v_H$), and R_{min} is a constant headway offset. The parameters α_1, α_2, τ_1, τ_2, R_{min} are preset constants.

2) Honda Algorithm: The Honda algorithm [19] uses the following warning criterion:

$$R_w = -2.2\cdot RR + 6.2 \qquad (2)$$

which is based on the *TTC*1 measure with a constant headway offset of 6.2 m. A warning is issued when the *TTC*1, after offset adjustment, is below 2.2 s.

The Honda overriding algorithm consists of two parts, depending on whether the lead vehicle is expected to stop within the considered time range τ_2. It is assumed that the lead vehicle applies constant braking at deceleration level $-\alpha_2$ (if the estimated lead vehicle stopping time $t_{LS} \equiv v_L/\alpha_2 < \tau_2$) or $-\alpha_1$ (if $t_{LS} \geq \tau_2$), while the host vehicle starts to brake after reaction time τ_1 at deceleration level $-\alpha_1$. Then R_o is estimated as the minimum range buffer needed to avoid collisions until τ_2 at both situations.

$$R_o = \begin{cases} v_H\tau_2 - \dfrac{\alpha_1}{2}(\tau_2-\tau_1)^2 - \dfrac{v_L^2}{2\alpha_2} & t_{LS} < \tau_2 \\ -RR\cdot\tau_2 + \alpha_1\tau_1\tau_2 - \dfrac{\alpha_1}{2}\tau_1^2 & t_{LS} \geq \tau_2 \end{cases} \qquad (3)$$

The parameters α_1, α_2, τ_1, τ_2 are constants.

3) Berkeley Algorithm: The Berkeley algorithm [20] proposes a conservative R_w to provide a wide range of visual feedbacks (cautionary warnings) to the driver, and a non-conservative R_o to reduce undesirable effects of overriding to normal driving operations. It is assumed that the lead vehicle brakes at the maximum constant deceleration level $-\alpha$, while the host vehicle starts to brake after reaction time τ at the same deceleration level. The R_w is estimated as the minimum range buffer needed to avoid collisions until both vehicles come to a full stop, while the R_o only considers the range buffer needed from time 0 to τ.

$$R_w = \frac{v_H^2 - v_L^2}{2\alpha} + v_H\tau + R_{min} \qquad (4)$$

$$R_o = -RR\cdot\tau + \frac{1}{2}\alpha\tau^2 \qquad (5)$$

The parameters α, τ, R_{min} are constants.

4) NHTSA Alert Algorithm: The NHTSA alert algorithm [6] considers slightly more complicated scenarios. It assumes that the lead vehicle decelerates at a constant rate a_L, while the host vehicle keeps a current constant rate of deceleration a_H for a reaction time t_r, after which it applies constant braking at the maximum deceleration level $a_{H_{max}}$ ($a_{H_{max}} \leq a_L < 0$). Two different situations are considered, depending on whether the lead vehicle stops first or the host vehicle stops first under the above assumptions. The lead vehicle stopping time t_{LS} and host vehicle stopping time t_{HS} are estimated by:

$$t_{LS} = -v_L/a_L \qquad (6)$$

$$t_{HS} = \begin{cases} t_r - \dfrac{v_H + a_H t_r}{a_{H_{max}}} & v_H + a_H t_r > 0 \\ -v_H/a_H & \text{otherwise} \end{cases} \qquad (7)$$

Usually it is assumed that $v_H + a_H t_r > 0$, in which case a warning system might be helpful. The range buffer needed to avoid collisions are computed by the following:

for $t_{LS} \leq t_{HS}$

$$R_w = v_H t_r + \frac{1}{2}a_H t_r^2 - \frac{(v_H + a_H t_r)^2}{2a_{H_{max}}} + \frac{v_L^2}{2a_L} + D_{thresh} \qquad (8)$$

for $t_{LS} > t_{HS}$ or $a_L \geq -1$

$$R_w = -RR\cdot t_r - \frac{1}{2}a_R t_r^2 + \frac{(RR + a_R t_r)^2}{2(a_L - a_{H_{max}})} + D_{thresh} \qquad (9)$$

where

$$D_{thresh} = 0.1\cdot v_H + 2. \qquad (10)$$

The relative acceleration ($a_R \equiv a_L - a_H$) is estimated from the time derivative of RR data measured by radar sensors, hence the a_L is no longer an assumed constant parameter here. The driver reaction time (t_r) is normally set to 1.5 s, and is reduced to 0.5 s when the brake is applied. The assumed $a_{H_{max}}$ is set to $-5.4\,\text{m/s}^2$ ($-0.55\,\text{g}$) for imminent alerts, and lower levels for cautionary alerts.

5) Other Alert Algorithms: Other alert algorithms have been developed for use in automotive collision warning and avoidance systems, as summarized in [21]. For example, if a_H is set to zero in the NHTSA alert algorithm, then Equation (8) simplifies to

$$R_w = v_H t_r - \frac{v_H^2}{2a_{H_{max}}} + \frac{v_L^2}{2a_L} + R_{min} \qquad t_{LS} \leq t_{HS} \qquad (11)$$

In addition, if the lead vehicle keeps a constant speed slower than the host vehicle, *i.e.*, $a_L = a_H = a_R = 0$, then Equation (9) simplifies to

$$R_w = -RR\cdot t_r - \frac{RR^2}{2a_{H_{max}}} + R_{min} \qquad v_L < v_H \qquad (12)$$

Furthermore, if the lead vehicle is stopped or stationary, *i.e.*, $v_L = 0$, then Equation (12) can be rewritten as

$$R_w = v_H\cdot t_r - \frac{v_H^2}{2a_{H_{max}}} + R_{min} \qquad v_L = 0 < v_H \qquad (13)$$

There are still other alert algorithms that are based on *TTC*1 ($-R/RR$), t_h (R/v_H), or a linear combination of the two:

$$R_w = -RR\cdot\tau_1 + v_H\cdot\tau_2 + R_{min} \qquad (14)$$

where τ_1 and τ_2 are predefined parameters as before.

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III. NEW CRITERION PROPOSAL

Most warning and overriding criteria used in automotive collision avoidance systems are expressed in terms of range. The measured current range R is compared with the warning range R_w or overriding range R_o to decide if warning or overriding is needed. It is difficult to clearly quantify the level of danger or threat from the comparison result since the range criteria vary nonlinearly under different dynamic conditions. For instance, a non-dimensional linear warning level w was proposed [20]:

$$w = \frac{R - R_o}{R_w - R_o} \tag{15}$$

This is not appropriate since it is known that the danger level does not have a linear relationship with the range criteria. Therefore it is desirable to have a new criterion that directly quantifies the danger or threat level of the current situation objectively as well as assesses the urgency level for the required evasive action, *e.g.*, braking. A new time-based measure is presented next for this purpose.

A. *Time-to-last-second-braking T_{lsb} Measure*

Time-to-last-second-braking (T_{lsb}), is a new time-based measure proposed for rear-end collision threat assessment. It is defined as the time remaining for the driver or the control system at the current situation to take the last extreme evasive action, *e.g.*, braking at the maximum level, to avoid a rear-end collision. It is calculated based on the assumptions that if the lead vehicle is decelerating, it will continue to do so uniformly at the current a_L until it comes to a full stop, and that the host vehicle also will maintain the current a_H until the last moment when it will be able to decelerate at the maximum deceleration level $a_{H_{max}}$ to avoid the collision. Therefore T_{lsb} estimates how long the host vehicle can maintain the current state until it must brake at the maximum level to just avoid a rear-end collision with the lead vehicle. T_{lsb} can be estimated from the following six state variables:

$$T_{lsb} = f(v_H, a_H, R, RR, a_R, a_{H_{max}}), \tag{16}$$

where the v_H and a_H can be measured by vehicle state sensors, the R and RR can be measured by on-board radar or lidar sensors, the a_R can be estimated from the RR history, and the $a_{H_{max}}$ can be estimated from tire-road friction coefficient monitor, as reviewed in [1].

The assumptions used here to calculate T_{lsb} are close to those used by the NHTSA alert algorithm. The difference is that the NHTSA alert algorithm assumes a fixed driver reaction time t_r and computes how much distance buffer is left at the current situation, while the T_{lsb} measure calculates how much time buffer is left for the driver or the control system to react in order to achieve the desired minimum distance buffer R_{min} during the collision avoidance process.

It follows from the definition of the new T_{lsb} measure that it gives a quantitative assessment of the current urgency and severity levels of the potential threats in terms of time, which are highly useful for threat assessment analysis for collision warning and avoidance systems.

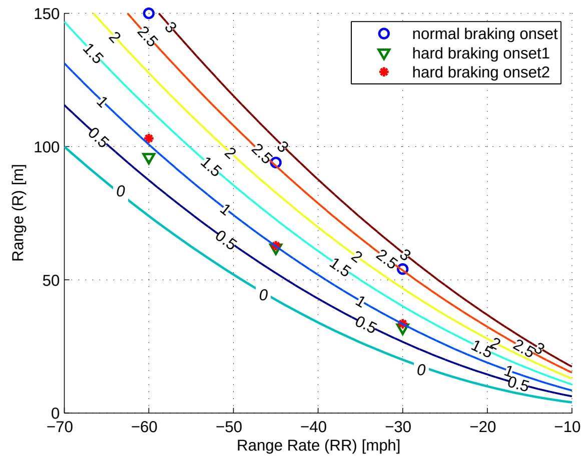

Fig. 1. T_{lsb} Contours in seconds with CAMP Data under Scenario 1: Host Vehicle Approaches Stopped or Slow Lead Vehicle ($a_L = a_H = 0$, $a_{H_{max}} = -5\,\text{m/s}^2$).

1) Scenario 1: First, a simple scenario is considered where the lead vehicle is initially stopped or traveling at a constant slower speed than the host vehicle (*i.e.*, $a_L = 0$, $RR < 0$). This is an important type of scenario where a collision avoidance system may be helpful. For instance, an inattentive driver might overlook a stopped or slowly moving vehicle ahead or underestimate its threat level until it is too late. The characteristic of this type of scenario is that the closing speed is usually high and often evasive action is necessary even when the range is still rather large. Hence it is important for the driver or the sensory system to detect an object and estimate the R & RR at a range of up to $150\,\text{m} \sim 200\,\text{m}$. The contours of T_{lsb} for this scenario are plotted as parabolic curves on the R *vs.* RR plot, shown in Figure 1.

The human drivers' last-second "normal" and "hard" braking onset data were recorded in Crash Avoidance Metrics Partnership (CAMP) experiments [3], [5], and the data for the lead vehicle stationary trials are also plotted in Figure 1 using different markers. These data points represent the average range for host vehicle braking onsets under different conditions, *i.e.*, last-second normal or hard braking conditions, and different initial host vehicle speeds $v_H = 30\,\text{mph}$, 45 mph, or 60 mph, respectively. It can be observed that the last-second normal braking data align nicely with the $T_{lsb} = 2.5\,\text{s}$ curve, which implies that alert drivers normally brake 2.5 s before the last moment when maximum braking is needed. Furthermore, two sets of CAMP last-second hard braking data both align well with the $T_{lsb} = 1\,\text{s}$ curve, which suggests that an attentive driver would perform a last-second hard braking action about 1 s before maximum (the hardest) braking is needed to avoid a rear-end collision. These alignments are especially good when the host vehicle speed is not too high (*e.g.*, $v_H = 30\,\text{mph}$ or 45 mph) and within a range of 100 m or so. This implies that human drivers have a fairly good sense of urgency about when to

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take a last-second evasive action under an attentive condition and a medium threat level, *e.g.*, when the host vehicle is approaching a red light or a car stopped at an intersection. Driver response timing appears to be highly consistent under these conditions.

Therefore the proposed T_{lsb} measure appears to reflect human drivers' sense of urgency to take the last evasive action, hence it is a good candidate for threat assessment analysis.

2) Scenario 2: In Scenario 2, the lead vehicle and the host vehicle initially travel at the same speed ($RR = 0$) with a certain initial time headway between them; then the lead vehicle suddenly starts to brake at constant a_L. This type of scenario is also important, since the sudden braking of lead vehicles on freeways is a major cause of traffic accidents [22]. The characteristic of this scenario is that usually the initial R is not too large (*e.g.*, $R < 50\,\mathrm{m}$) and the requirement on the driver or the sensory system to detect an abrupt negative change in RR and a_R is high.

The human drivers' last-second "normal" and "hard" braking onset data recorded during the lead vehicle decelerating trials in CAMP experiments [3], [5] can be used to estimate the T_{lsb} measure under Scenario 2. For more details, please refer to [23].

It is interesting to note that $T_{lsb} \approx 0.5\,\mathrm{s}$ for all of the last-second hard braking data under the heavy lead vehicle braking scenario ($a_L = -3.8\,\mathrm{m/s^2}$), while $T_{lsb} \in [1, 1.5]\,\mathrm{s}$ for the corresponding last-second normal braking data, implying the urgency and severity of this kind of scenario. In addition, the time buffer left until last-second braking increases as the lead vehicle deceleration level decreases and/or the host vehicle speed increases.

B. Estimation of the T_{lsb} Measure

1) Computing T_{lsb}: In general, as with the NHTSA alert algorithm described in Section II-C.4, two different cases are considered to estimate T_{lsb}, depending on whether the lead vehicle is expected to stop first or not. The t_{LS} is still estimated by Equation (6), while the t_{HS} estimation changes to the following, since it now depends on the T_{lsb} instead of t_r:

$$t_{HS} = T_{lsb} - \frac{v_H + a_H T_{lsb}}{a_{H_{max}}} \qquad v_H + a_H T_{lsb} > 0 \qquad (17)$$

Generally it is assumed that the condition $v_H + a_H T_{lsb} > 0$ holds, otherwise the host vehicle is already decelerating hard enough, hence no further action need be taken.

Accordingly, Equations (8) and (9) also change to the following:

for $t_{LS} \leq t_{HS}$

$$R = v_H T_{lsb} + \frac{a_H}{2} {T_{lsb}}^2 - \frac{(v_H + a_H T_{lsb})^2}{2a_{H_{max}}} + \frac{v_L^2}{2a_L} + R_{min} \quad (18)$$

for $t_{LS} > t_{HS}$

$$R = -RR \cdot T_{lsb} - \frac{1}{2} a_R {T_{lsb}}^2 + \frac{(RR + a_R T_{lsb})^2}{2(a_L - a_{H_{max}})} + R_{min} \quad (19)$$

TABLE III

INPUT NOISE DISTRIBUTIONS.

Parameter	Noise Distribution
v_H	$U[-0.15, 0.15]$
a_H	$G(-0.07, 0.17)$
R	$G(0.4, 0.025)$
RR	$U[-0.0625, 0.0625]$
a_R	$G(-0.6, 0.1)$

TABLE IV

TRUE INPUT DISTRIBUTIONS.

Parameter	Scenario 1	Scenario 2
v_H	$U[20, 30]$	$U[20, 30]$
a_H	$L(0, 0.3)$	$L(0, 0.3)$
R	$U[60, 80]$	$U[20, 40]$
RR	$U[-v_H, -v_H + 5]$	$U[-v_H + 20, -v_H + 30]$
a_R	$L(-a_H, 0.3)$	$L(-5 - a_H, 0.3)$

T_{lsb} can be solved from the equations above, depending on the current conditions. In practice, first it is assumed that the lead vehicle stops first ($t_{LS} \leq t_{HS}$), then T_{lsb} can be solved from Equation (18), and then t_{HS} can be computed from Equation (17), and finally whether $t_{LS} \leq t_{HS}$ holds or not can be verified. If it holds, then the computation for T_{lsb} is complete. Otherwise T_{lsb} is solved from Equation (19), where the more positive solution is taken and the other solution discarded.

2) Error Estimation of T_{lsb}: The error of the estimated T_{lsb} depends on the error or measurement noise of the six underlying state variables as specified in Equation (16). For simplicity, it is assumed that the input measurement noises are independent random variables with the distributions given in Table III [6]. Here $U[a, b]$ represents a uniform distribution in the interval from a to b, while $G(\mu, \sigma)$ represents a Gaussian distribution with mean μ and std σ. All units are SI unless otherwise noted. These noise distributions were derived from a noise analysis of data collected from the prototype collision warning system in the Engineering Development Vehicle (EDV) developed under the Automotive Collision Avoidance System field operational test (ACAS FOT) [6].

In order to estimate the error of T_{lsb}, the assumed true input parameters under different scenarios are drawn randomly using the distributions specified in Table IV, where $L(\mu, \sigma)$ represents the Laplacian distribution with mean μ and std σ. In addition, the true maximum available host vehicle deceleration $a_{H_{max,true}}$ is sampled from a truncated Gaussian distribution with mean $-5.9\,\mathrm{m/s^2}$, std $1\,\mathrm{m/s^2}$, minimum $-7.8\,\mathrm{m/s^2}$, and maximum $-2.9\,\mathrm{m/s^2}$. It is assumed that $a_{H_{max}}$ can be estimated within $\pm 10\%$ white noise.

The true time-to-last-second-braking ($T_{lsb,true}$) can be calculated based on the true inputs sampled from the distributions shown in Table IV, and the estimated value ($T_{lsb,est}$) can be calculated based on the corresponding noisy sensor inputs data, which are obtained by adding sensor measurement noise (drawn from the distributions shown in Table III) to the true inputs. Then the error distribution of the T_{lsb} measure (*i.e.*, $T_{lsb,est}$ - $T_{lsb,true}$) can be estimated by repeating the above

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TABLE V

ERROR OF T_{lsb} ESTIMATION DUE TO SENSOR NOISE IN SECONDS.

	0.1%	1%	50%	99%	99.9%	mean	std
S1	-1.06	-0.80	-0.26	0.16	0.24	-0.27	0.21
S2	-0.81	-0.66	-0.27	0.03	0.10	-0.28	0.16

calculations for a large number of trials and normalizing the data. The various percentile values and statistical measures of the T_{lsb} estimation error under both Scenario 1 (S1) and 2 (S2) are summarized in Table V. It can be noted that, under the current assumptions, 99% of the T_{lsb} estimation error is within 1 s, and that the estimated T_{lsb} value does not exceed the true value by more than 0.25 s with a probability of over 99.9%.

C. Warning/Overriding Criteria in terms of T_{lsb}

The T_{lsb} measure provides a straightforward and quantitative threat assessment of the current dynamic situation. From its definition it follows that potential collisions would be avoided if the driver or the control system could react within T_{lsb} with a sufficient level of deceleration.

From previous work on driver reaction times as reviewed in Section II-B, human drivers usually do not have a consistently quick response on the road. It may take up to 2 s to account for 90% of drivers' reaction times under natural driving scenarios without any warning signals. The situation is slightly better in that 90% of drivers can react within 1.8 s if a visual warning signal is used, 1.55 s if an auditory warning signal is issued, and 1.35 s if visual plus auditory warning signals are applied. However, the interference level of the warning signals also increases (from none, visual, auditory, to visual + auditory signals) as the driver reaction time decreases. The higher the interference level, the more it is probable that drivers would experience the warning signals as a nuisance. Hence it is desirable to set the warning timing not too early to reduce the interference level, and at the same time not too late to give most drivers sufficient time to react. As a result of this trade-off it is difficult to achieve a satisfactory performance if the collision avoidance system solely relies on human drivers to take action in an emergency, due to the significant variations in driver behavior.

To overcome human driver limitations, an overriding system can be used at critical moments to automatically apply braking at the maximum level to avoid collisions. The advantages are that an override system is not subject to the influence of driver reaction times and braking level variability, and that the T_{lsb} measure can give a relatively accurate estimate of how much time is left for the overriding system to react.

Based on the above discussions and observations, the following warning and overriding criteria in terms of the T_{lsb} measure are proposed:

- $1.5\,\text{s} \le T_{lsb} < 2.5\,\text{s}$: Cautionary warning (*e.g.*, visual signal)
- $0.5\,\text{s} \le T_{lsb} < 1.5\,\text{s}$: Imminent warning (*e.g.*, visual + auditory signals)
- $T_{lsb} < 0.5\,\text{s}$: Overriding (automatic braking)

The overriding threshold (0.5 s) is chosen to avoid collisions with a probability of over 99.9%, according to the T_{lsb} error distribution described in Section III-B.2 and assuming the system delay of automatic braking to be 0.25 s. The CAMP data shown in Section III-A also suggest that alert drivers in most situations would have taken a normal or hard braking action before the T_{lsb} drops down to 0.5 s.

The two one-second warning stages are defined according to general human driver reaction times and the error distribution of T_{lsb} estimation. The warning thresholds can be further adjusted according to each individual driver's sensitivity level to warnings. For instance, a responsive driver might desire shorter warning time ranges than a slow driver. The performance of the proposed T_{lsb} warning and overriding criteria are analyzed in terms of miss and false alarm probabilities in [23].

The proposed criteria have several advantages over the previous algorithms reviewed in Section II-C. First, they are defined in the time domain instead of distance domain, which is in agreement with natural human sense and judgment of urgency. Additionally, T_{lsb} provides a concrete measure of the amount of time that is left for the driver or the control system to react to avoid a potential collision, and therefore can serve as a direct measure of the urgency and severity of the threat of collision.

Second, the estimation process of the T_{lsb} measure takes into account all possible current information characteristic of the current dynamic situation (*i.e.*, $v_H, a_H, R, RR, a_R, a_{Hmax}$), while most previous algorithms only use partially updated information and assume that the rest of the state variables remain constant. It follows that the estimation of T_{lsb} will be more sensitive to real-time sensor noise and that the accuracy of T_{lsb} estimates can be improved by increasing the reliability and precision of sensor measurements. However, even when the sensor data are noisy, T_{lsb} still produces better threat estimates than a constant assumption in most cases.

Third, the T_{lsb} criterion is less sensitive to human driver variability. In contrast to previous algorithms, the computation of the T_{lsb} measure no longer depends on an assumed human driver reaction time, even though the warning criteria are still established with reference to the human driver reaction times. The overriding criterion depends on neither human driver reaction time nor braking behavior, which are two important human factors in other warning/overriding algorithms.

Fourth, the overriding system can avoid collisions more effectively at the last moment based on the T_{lsb} measure. As mentioned above, the overriding system is able to avoid rear-end collisions with a probability of over 99.9% under the current assumptions.

IV. COMPARISONS

In this section, the first four warning and overriding algorithms described in Section II-C are compared with the

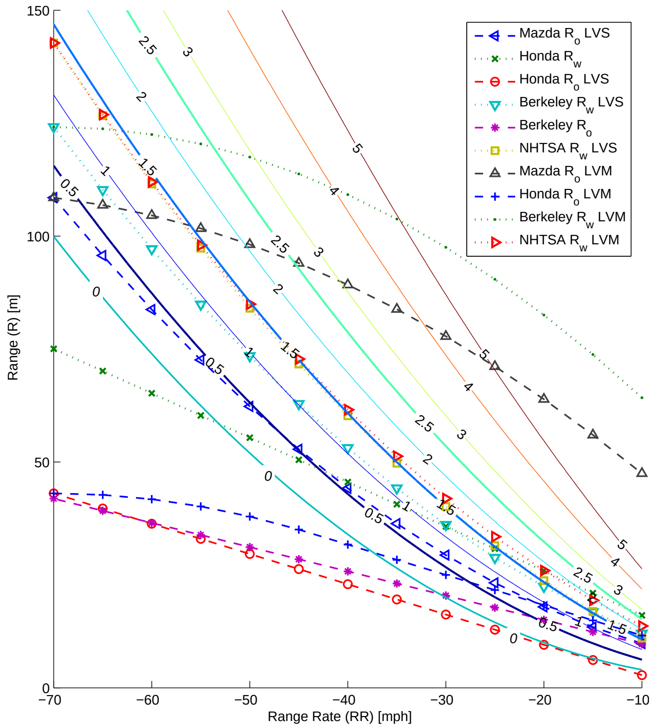

Fig. 2. T_{lsb} Contours (solid lines) in seconds with Various Warning (R_w, dotted lines) and Overriding (R_o, dashed lines) Boundary Curves under Scenario 1: Host Vehicle Approaches Stopped or Slow Lead Vehicle ($a_L = a_H = 0$, $a_{Hmax} = -5\,\text{m/s}^2$).

proposed T_{lsb} warning and overriding criteria under two scenarios.

A. Comparison under Scenario 1

As mentioned in Section III-A.1, the T_{lsb} contours can be plotted as parabolic curves on a R *vs.* RR plot, as shown in Figure 1. In Figure 2, the T_{lsb} contours are plotted again with thick curves representing the warning and overriding thresholds proposed in Section III-C, *i.e.*, $T_{lsb} = 0.5\,\text{s}$, $1.5\,\text{s}$, and $2.5\,\text{s}$.

In contrast, other algorithms have different warning or overriding thresholds for lead vehicle stopped (LVS) scenario and lead vehicle moving (LVM) scenario, respectively, as shown in Figure 2. For the LVS scenario ($v_L = 0$), the Mazda overriding curve is close to the $T_{lsb} = 0.5\,\text{s}$ curve, the Berkeley warning curve is roughly close to the $T_{lsb} = 1\,\text{s}$ curve, and the NHTSA warning curve is close to the $T_{lsb} = 1.5\,\text{s}$ curve. Note that these three curves are all concave curves like the T_{lsb} contours. While the Honda warning/overriding and the Berkeley overriding thresholds are straight lines on the R *vs.* RR plot, implying that they are only based on the *TTC*1 measure with a possible constant distance headway offset adjustment.

For the LVM scenario, only the threshold curves with $v_H = 70\,\text{mph}$ are plotted in Figure 2, hence the LVS and LVM curves for the same algorithm intersect at $RR = -70\,\text{mph}$. Note that all LVM threshold curves are convex curves except for the NHTSA warning curve. This is because they all assume certain *constant* lead vehicle deceleration

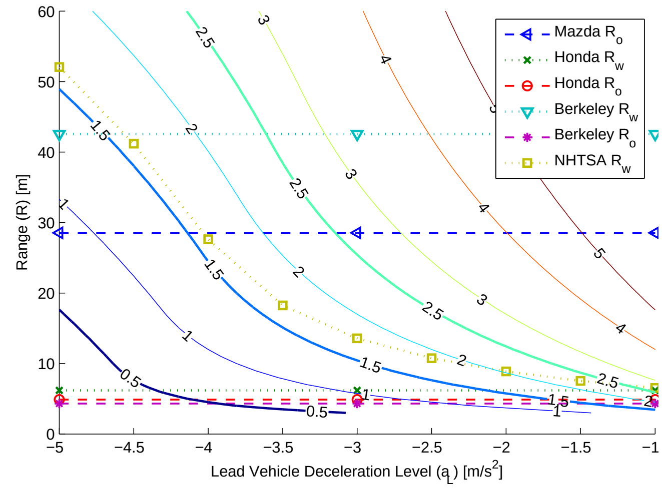

Fig. 3. T_{lsb} Contours (solid lines) in seconds with Various Warning (R_w, dotted lines) and Overriding (R_o, dashed lines) Boundary Curves under Scenario 2: Lead Vehicle just Starts to Decelerate, where both vehicles initially travel at the same speed 70 mph ($a_H = 0$, $a_{Hmax} = -5\,\text{m/s}^2$).

level a_L while the NHTSA alert algorithm uses the *current* a_L values estimated in real time. However, when v_L is close to v_H, the possibility of sudden braking of the lead vehicle must be considered even if the current $a_L = 0$, which converts to Scenario 2, discussed below.

Furthermore, it can be observed from Figure 2 that the NHTSA and the proposed T_{lsb} warning criteria are the most conservative under the LVS scenario, giving drivers sufficient warnings. It is also noted that some warning and overriding thresholds even fall below the $T_{lsb} = 0$ curve, implying that a deceleration level higher than $-5\,\text{m/s}^2$ is needed to avoid collisions in these situations. In particular the Honda and Berkeley overriding algorithms require a deceleration level of more than 1 g ($-9.8\,\text{m/s}^2$) at the time of overriding when $RR = -70\,\text{mph}$, which is too large for collision avoidance, even though they may still assist with collision mitigation.

B. Comparison under Scenario 2

When the lead vehicle decelerates ($a_L < 0$), the situation becomes more complex with multiple possibilities under different kinematic conditions. A typical case is considered here where the lead vehicle and the host vehicle travel at the same speed (*e.g.*, $v_L = v_H = 70\,\text{mph}$) initially, and then the lead vehicle suddenly starts to brake at a constant deceleration level $a_L < 0$. At the moment the lead vehicle starts to brake (*i.e.*, $RR = 0$ still holds), the T_{lsb} contours can be plotted in terms of R and a_L, as shown in Figure 3. Again the thick curves represent the T_{lsb} warning and overriding thresholds proposed in Section III-C, *i.e.*, $T_{lsb} = 0.5\,\text{s}$, $1.5\,\text{s}$, and $2.5\,\text{s}$.

The first four warning and overriding algorithms described in Section II-C are also plotted in Figure 3 for comparison. Since all previous algorithms except for the NHTSA do not take current a_L into account, they are all plotted as horizontal lines in Figure 3, while the NHTSA warning curve is still close to the $T_{lsb} = 1.5\,\text{s}$ curve as expected, since it

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assumed that $t_r = 1.5\,\mathrm{s}$ in addition to the assumptions used by the T_{lsb} measure. It can be observed from the figure that when the lead vehicle brakes lightly (*e.g.*, $a_L > -3\,\mathrm{m/s^2}$), the Berkeley warning and Mazda overriding thresholds are generally too conservative since heavier a_L was assumed in these algorithms, while the Honda warning/overriding and Berkeley overriding systems might act too late when the lead vehicle brakes hard (e.g., $a_L < -3\,\mathrm{m/s^2}$).

More different dynamic situations are discussed in [23]. From the comparisons above it can be inferred that it is important for a collision avoidance system to take the lead vehicle deceleration (a_L) information into account to make better threat assessments of the current situation. Even if only a rough and slightly delayed estimate of a_L is available due to noisy sensor data, it would still be better than a constant assumption in most cases. In addition, when the lead vehicle is not braking heavily ($a_L > -3\,\mathrm{m/s^2}$), the possibility of sudden heavy braking of the lead vehicle in the near future must be considered, confirming the need for regular updates to a_L, and a threat assessment measure such as T_{lsb} that can incorporate the changes in a_L.

V. CONCLUSIONS

A new threat assessment measure, time-to-last-second-braking (T_{lsb}) is proposed, and its advantages over previous measures are discussed. It directly quantifies the threat level of the current dynamic situation objectively as well as assesses the urgency level for the required evasive action, *e.g.*, braking. It is also in agreement with human natural judgment of the urgency and severity of threats. Furthermore, new warning and overriding criteria are proposed based on the new T_{lsb} measure, which is least affected by driver behavior variability. The new criteria characterize the current dynamic situations better than the previous criteria, providing more appropriate warning and more effective overriding at the last moment.

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