J. Biomechanics Vol. 18, No. 1, pp. 39–47, 1985.
Printed in Great Britain

0021–9290/85 $3.00 + .00

THE FREQUENCY CONTENT OF GAIT

ERIK K. ANTONSSON* and ROBERT W. MANN

Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, U.S.A.

Abstract—We address amplification of noise in double differentiation of position histories for dynamic analysis of gait. Measurements of the frequency domain characteristics of signal and noise are required to quantitatively assess errors in raw, filtered, and dynamic gait data. The results of a simple technique to determine the frequency content of gait using a population of 12 subjects and a total of 30 gait records is presented.

INTRODUCTION

All measurement signals are contaminated with noise to a greater or lesser degree. The problem is particularly severe in the measurement of the kinematics of gait due to the inherent complexity of the biomechanical system and nuances of its movement patterns, and also due to the data acquisition and reduction techniques associated with practical systems for gait analysis. The problem is exacerbated when kinematic data is to be used for dynamics, since practically all gait analysis approaches acquire position information which must be subjected to double differentiation in order to produce acceleration signals for dynamic analysis. Inherently noisy position (or rotation) information so processed, greatly exaggerates the noise present, and means must be adopted to 'smooth' the processed data. The general technique is to apply a low-pass filter or its equivalent. For example Cappozzo *et al.* (1975) use the first five terms of a Fourier series to describe the frequency content of gait; Winter *et al.* (1974) apply a 5 or 6 Hz, low-pass filter; Soudan and Dierckz (1979) use spline functions, and Alexander and Jayes (1980) use five terms of a Fourier series. However, without explicit knowledge of gait signal† frequency content such methods introduce unknown errors which include the extent of noise contaminating the signal below the low-pass filter cutoff, and possible elimination of that portion of the gait signal contained above the filter cutoff frequency. Better information on the frequency domain characteristics of the unambiguous gait signal would permit the determination of the quality of raw, filtered, and double differentiated gait data.

MOTIVATION

We propose and demonstrate a direct approach to measuring the frequency content of gait. However, first we will quantify the effect of contaminating noise on measured signals in general. Double differentiation of position histories amplifies noise in the signal by the square of its frequency. If noise (N) characterized by

$$N = (A_n)\sin(\omega_n)t \tag{1}$$

(where: A_n is an amplitude, ω_n is a frequency and t is time) is added to a signal, after double differentiation the noise becomes

$$\mathrm{d}^2N/\mathrm{d}t = -A_n(\omega_n^2)\sin(\omega_n)t \tag{2}$$

and attenuating noise at frequency ω_n becomes vital.

A graphical illustration of the influence of noise is given in Fig. 1 which shows how noise amplitude relative to signal amplitude varies with frequency as a consequence of the double differentiation of equation (2). Assume the signal is a sinusoid (S) with an amplitude of 100 units and a frequency of 1 Hz (the approximate fundamental frequency of normal gait).

$$S = 100\sin(2\pi t). \tag{3}$$

Superimpose noise (N) at different frequencies (F_n) according to

$$N = A_n\sin(F_n 2\pi t). \tag{4}$$

Double differentiation of both equations (3) and (4) produces

$$\mathrm{d}^2S/\mathrm{d}t = -100(2\pi)^2\sin(2\pi t) \tag{5}$$

$$\frac{\mathrm{d}^2N}{\mathrm{d}t} = -A_n(F_n 2\pi)^2\sin(F_n 2\pi t). \tag{6}$$

For a signal-to-noise ratio (S/N) of the acceleration signals equal to 1 we equate the absolute value of the amplitudes of equations (5) and (6)

$$100(2\pi)^2 = A_n(F_n 2\pi)^2 \tag{7}$$

thus

$$A_n = 100/(F_n)^2. \tag{8}$$

A_n is plotted as a function of F_n in Fig. 1 as the iso-signal-to-noise contour. The ordinate of Fig. 1 can be read directly as that input noise amplitude, expressed as percent of the 1 Hz signal amplitude, which will produce a signal to noise ratio of 1 in the dynamics. Note that 3 Hz noise whose amplitude is only 10% of the raw 1 Hz signal amplitude will produce a noise

Received August 1982; *in revised form July* 1984.

*Current address: Division of Engineering and Applied Science, California Institute of Technology, Pasadena, CA 91125, U.S.A.

†A gait 'signal' is any time varying parameter of gait, such as the global position of a point on the femur as a function of time, or the force between the foot and the floor during stance.

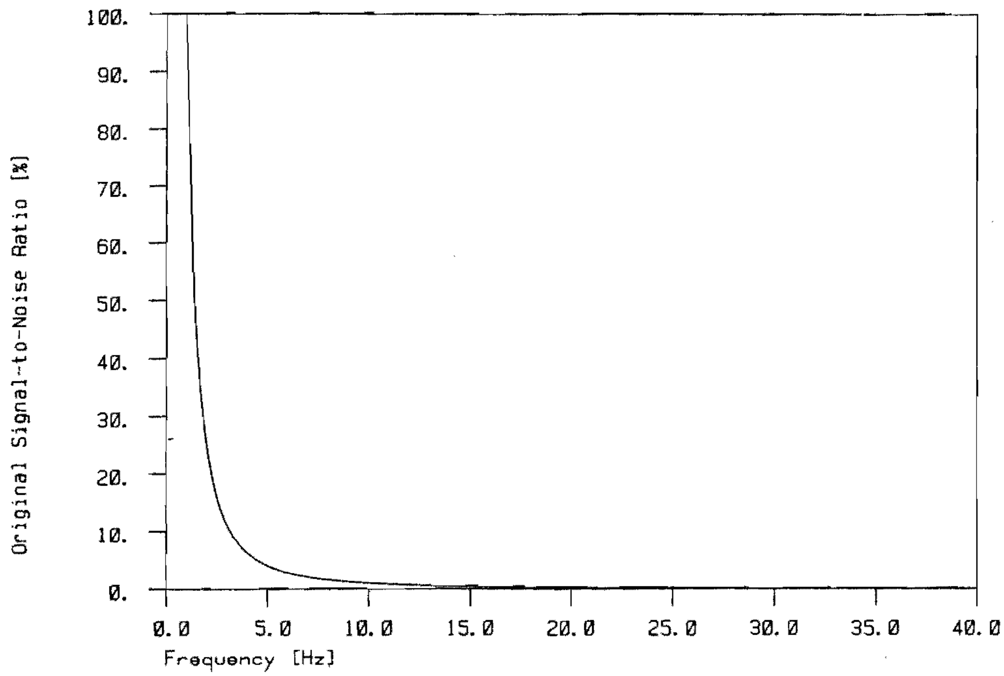

Fig. 1. Iso-Signal-to-Noise Contour for a double differentiated 1 Hz, 100 unit original amplitude signal.

amplitude equal to the signal following the double differentiation; only 1 % noise achieves a $S/N = 1$ at 10 Hz; whereas 0.1 % noise produces the same result at 32 Hz. Thus Fig. 1 constrains the additive noise in the signal to amplitudes well below this 'iso-noise' curve if the gait data analyzed is to be valid.

The effects of this noise can be mitigated through the use of a filtering technique, but only if two conditions are met. First: that the noise frequencies do not overlap the signal frequencies, and second: that the data is sampled at more than twice the highest frequency in the composite signal and noise to avoid aliasing. Aliasing (the appearance of data at incorrect frequencies in the reconstructed discrete-time data) occurs when components of either signal or noise are present at frequencies higher than twice the sampling frequency. 'To be able to describe (a function) $f(t)$ exactly, it is necessary to sample $f(t)$ at a rate greater than twice its highest frequency' (Stearns, 1975, p. 37).

METHOD

Now that the importance of both the signal and noise frequency content have been quantified we will establish, on an instrument whose natural frequency is very much higher than any possible significant gait component and which also posesses a high signal to noise ratio, the time and frequency domain response of that portion of the gait cycle where the most abrupt and rapid position changes with time occur, thereby encompassing the 'worst case' accelerations in the biomechanical system. These accelerations occur at the foot during heel strike. Since kinematic accelerations are directly related to forces, a forceplatform can be employed to capture the frequency content of gait. Light *et al.* (1980), Mizrahi and Suzak (1982) and Voloshin *et al.* (1981) demonstrate that the frequency spectrum measured on a forceplatform with bare feet contains higher frequency components than do motions acquired elsewhere on the body. By employing this objective measure of the broadest possible spectrum of frequency components in the gait cycle with an instrument of known signal-to-noise ratio an accuracy specification can be determined for dynamic estimation as a function of cut-off frequency of the low-pass filter to be subsequently applied to kinematic gait analysis data. The approach and results we describe were motivated by our development of an automatic and precise computer-mediated opto-electronic mobility analysis system (Antonsson, 1982; Antonsson and Mann, 1983, 1979; Mann and Antonsson, 1983; Mann *et al.*, 1982).

EQUIPMENT AND PROCESSING

To measure foot–floor reaction forces a Kistler 9281A multi-component forceplatform is employed in our mobility analysis system. Analog voltage signals from the 8 piezo-electric-crystal charge amplifiers are converted to digital signals via a 12 bit multiplexed analog-to-digital converter subsystem located at the charge amplifiers, resulting in a resolution of the force platform output of 1 part in 4096. Measured noise amplitude of the forceplatform is less than ± 1 bit, thus

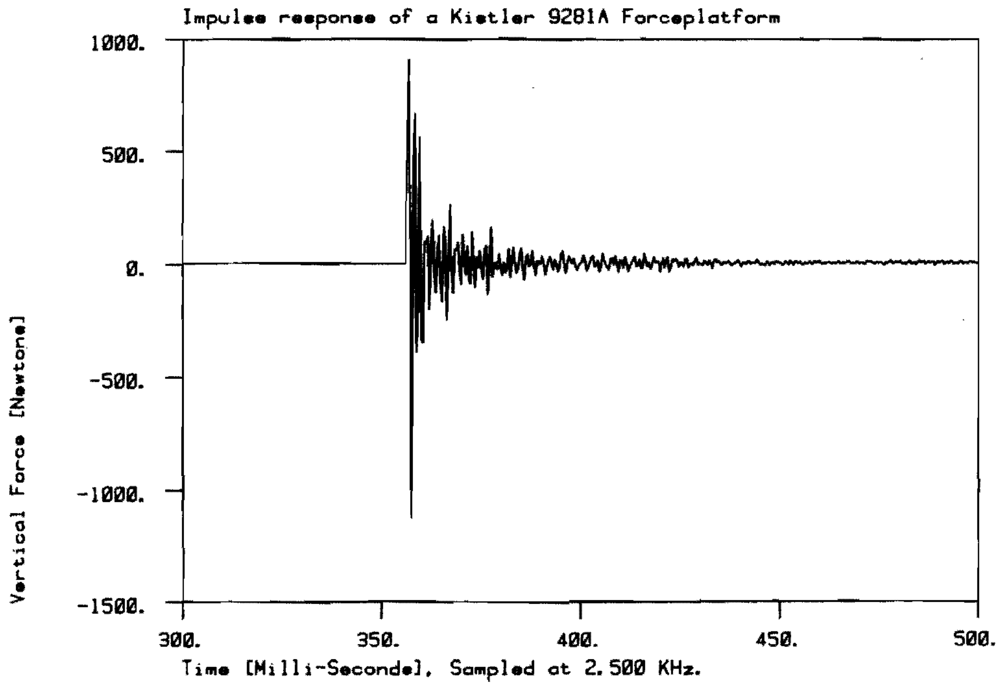

Fig. 2. Unloaded forceplatform impulse time response.

an overall signal-to-noise ratio of 2000 is achieved.* The manufacturer publishes a fundamental frequency of resonance for the forceplatform higher than 1.0 kHz. To check this we dropped a 30 mm diameter stainless steel ball 200 mm vertically onto the center of the forceplatform surface. The unfiltered analog output from the transducers was digitally sampled at 2.5 kHz. Figure 2 gives the time domain impulse response of the forceplate and Fig. 3 gives the frequency spectrum of the Fast Fourier Transform (FFT) of the data in Fig. 2. The results suggest that the fundamental frequency of the unloaded forceplate, at least for our installation, is about 700 Hz.

A similar experiment was conducted with the forceplatform loaded with a stationary human subject. The frequency domain impulse response is given in Fig. 4 and shows a change in the distribution of frequency content, but a very similar form to the unloaded case. The significant peaks are between 650 and 850 Hz, as expected for a 72 kg subject (connected by soft tissue) on a 30 kg platform with an equivalent supporting transducer stiffness of approximately 15 MN m^{-1}.

*In an earlier configuration of the forceplatform digitizing system the analog signals were carried some 20 m to the computer where they were digitized. Electro-magnetic interference in the laboratory environment degraded the overall signal to noise ratio of the forceplatform output to 10, dramatically demonstrating the higher fidelity achieved in digital signal transmission. In the present system the analog signal lines are only a few cm long.

To be certain that signals were sampled and analyzed correctly, a 30 Hz sine-wave from an analog voltage signal generator was applied to the inputs of the analog-to-digital (A/D) converter subsystem, substituting for one of the forceplate charge amplifier output voltages. Data were acquired at 2.0 kHz. Figure 5 plots the data as accumulated. Figure 6 shows the frequency domain results verifying that signals applied to the A/D converter were sampled correctly and that the routine produced correct results.

EXPERIMENTAL RESULTS

A population of twelve different subjects walked unshod across the plate at their natural stride length and cadence rendering a total of 30 stance phase data records of which Fig. 7 is typical. All gait data were sampled at 2.0 kHz. Figure 8 shows a time expansion of the sharp peak centered at about 165 milliseconds. The time domain data from Fig. 7 exhibits the frequency spectrum of Fig. 9. The FFT produces amplitude as a function of frequency from 0.0 (zero) Hz to 1.0 kHz. Only the lowest 100 Hz are plotted here because all results above that frequency are essentially zero. Since we know the resonant frequency of the forceplate is at least 700 Hz, and the FFT results for gait are monotonically decreasing from 100 to 1.0 kHz, clearly all of the frequency components of the highest acceleration component of normal gait have disappeared completely above 100 Hz. Therefore Fig. 9 is

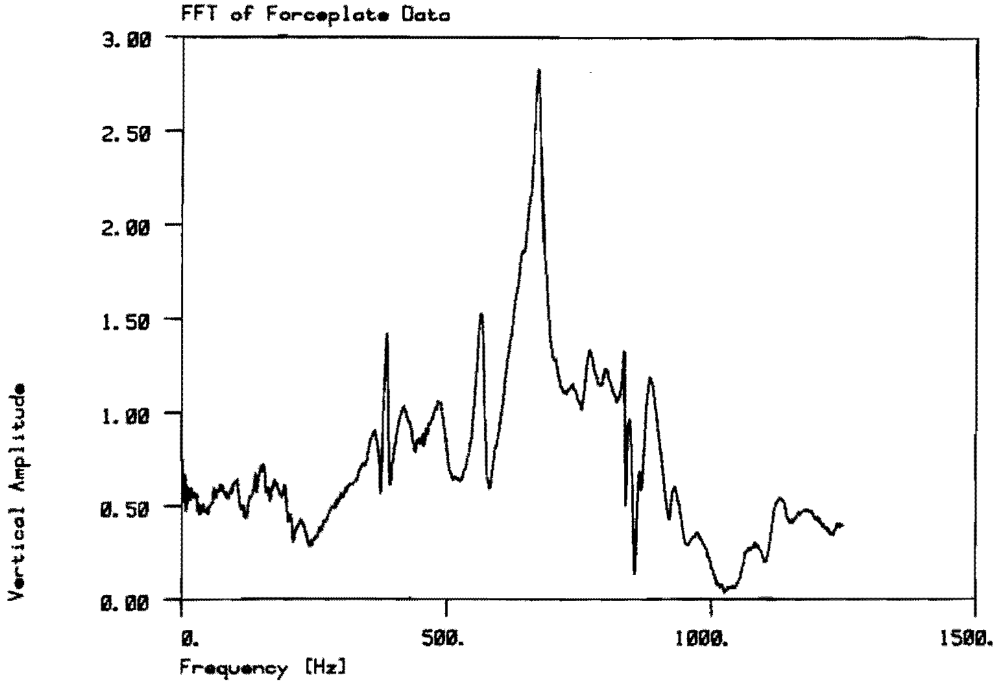

Fig. 3. Unloaded forceplatform impulse frequency response.

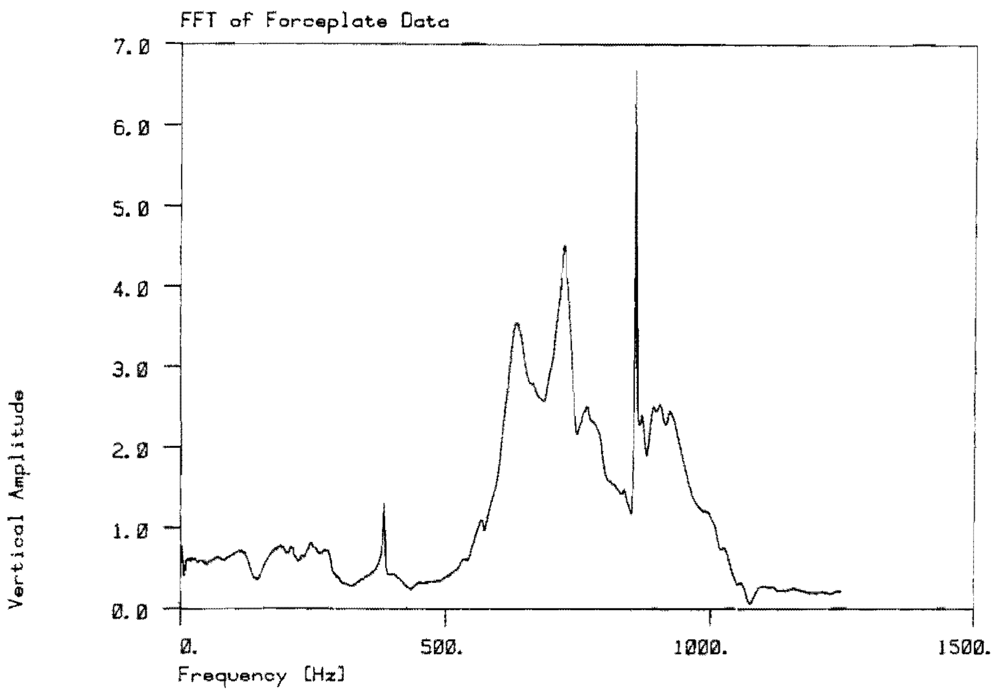

Fig. 4. Impulse frequency response of the forceplatform loaded with a human subject.

the frequency spectrum of the uppermost bound of the significant frequency components for one trial of normal human walking.

In order to obtain a useful *composite* frequency domain plot for all 30 experiments, each individual force record was normalized to the maximum force before the FFT was taken and then aggregated after. Results of the normalized composite FFT are plotted in percent of maximum amplitude in Fig. 10. Figure 11 shows the integral of *power* in the composite vertical force envelope plotted as percent of the total power contained between zero (0) Hz and 1.0 kHz.

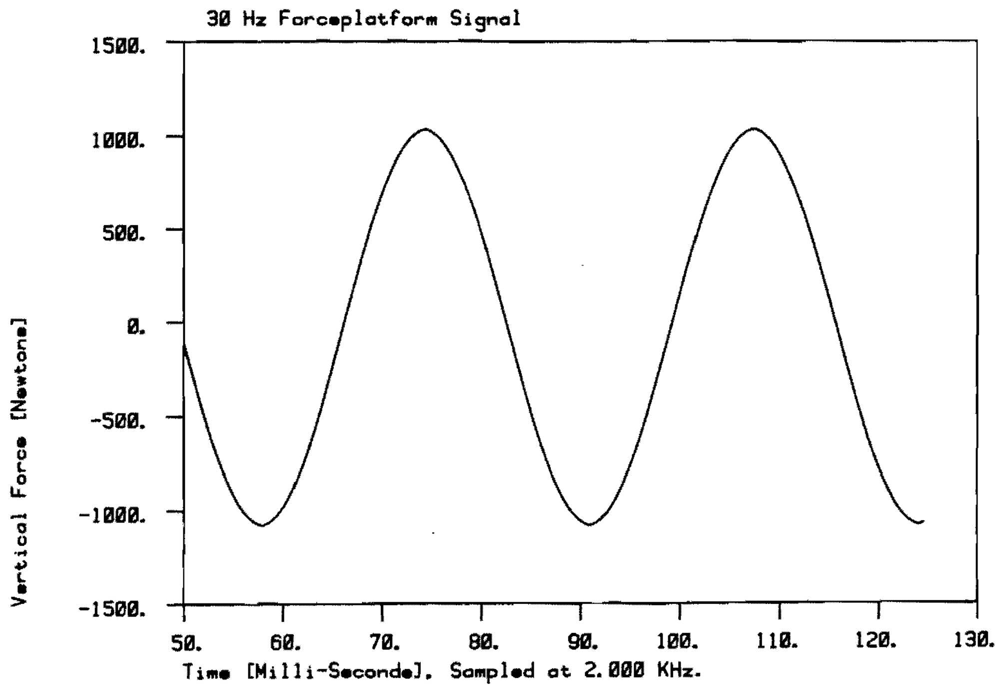

Fig. 5. Sampled 30 Hz sine-wave.

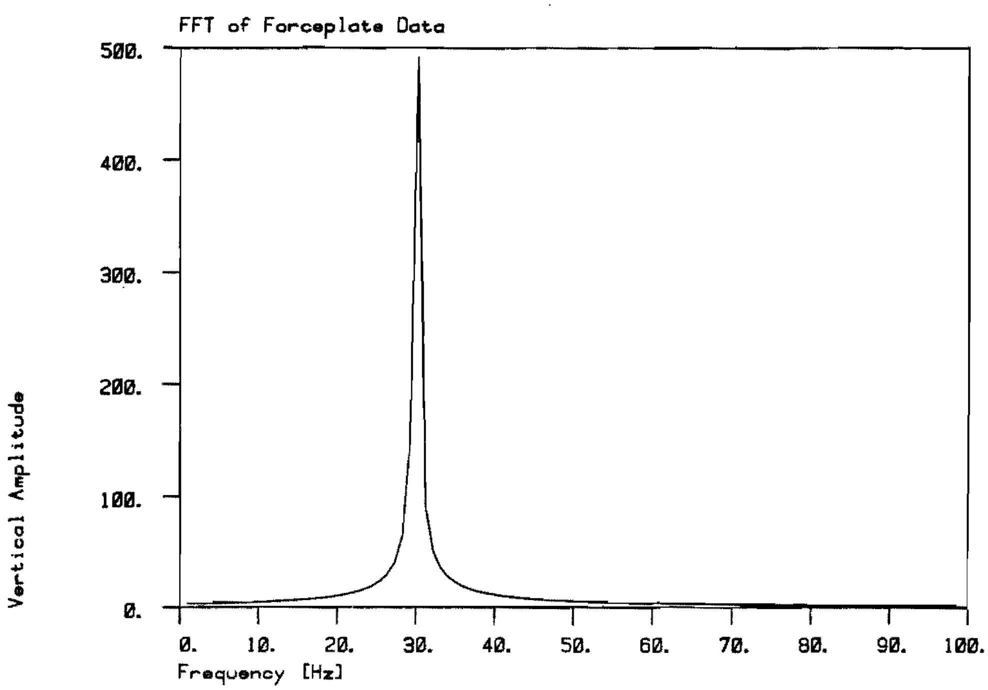

Fig. 6. FFT of sampled 30 Hz sine-wave.

CONCLUSIONS

Given the signal-to-noise ratio of our forceplatform measuring system and the true and complete frequency spectrum of the most abrupt component of human walking, it is instructive to develop a relationship which objectively relates the highest frequency included in the analysis to the required accuracy of the end-result dynamic estimates. The 'tail' of the composite frequency bound provides this relationship;

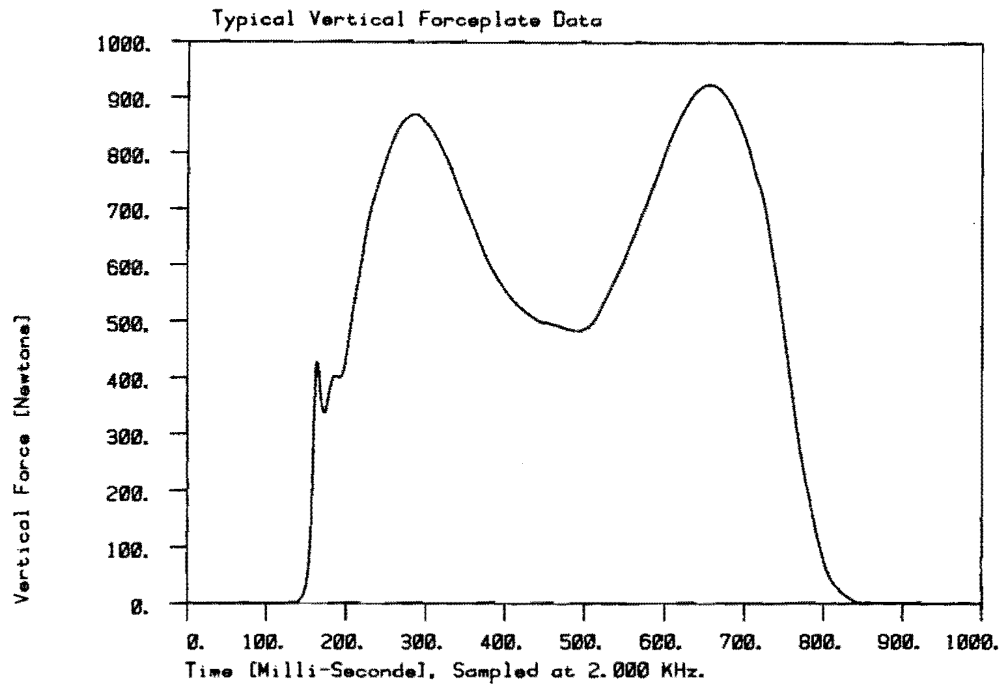

Fig. 7. Typical normal gait vertical force record.

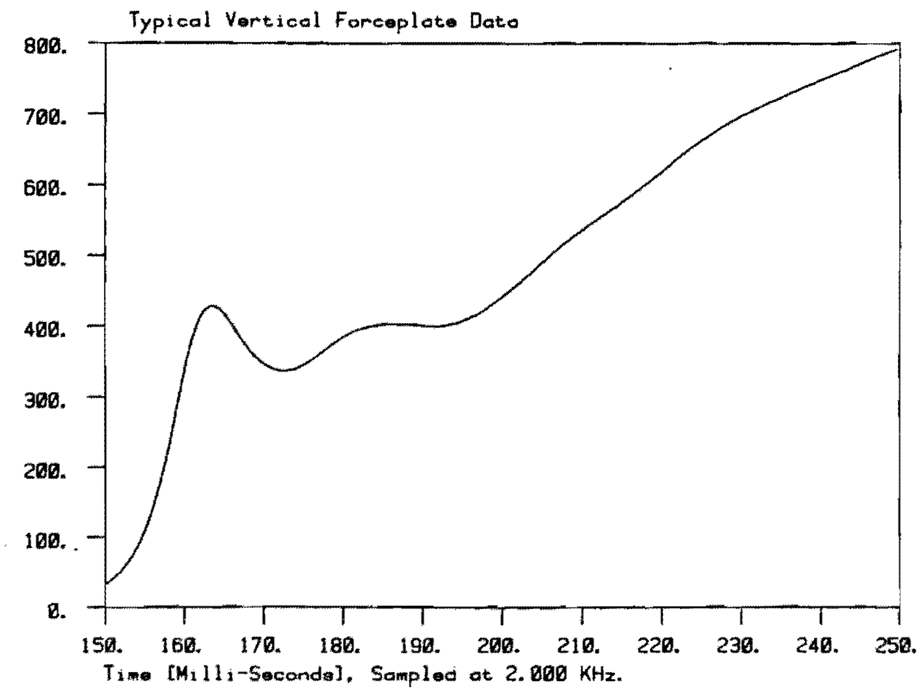

Fig. 8. Expanded region about heel strike for normal gait.

representative values can be read from Fig. 12, which is an expansion of the region between 10 and 100 Hz from Fig. 10. No amplitudes greater than 5 % of the fundamental exist above 10 Hz, none greater than 2 % above 20 Hz, and all amplitudes greater than 1 % are contained below 50 Hz. Figure 11 shows that 98 % of the power is contained below 10 Hz, and 99 % below 15 Hz. This information must be tempered by the noise characteristics of the particular gait analysis system being considered to achieve a trade-off between noise

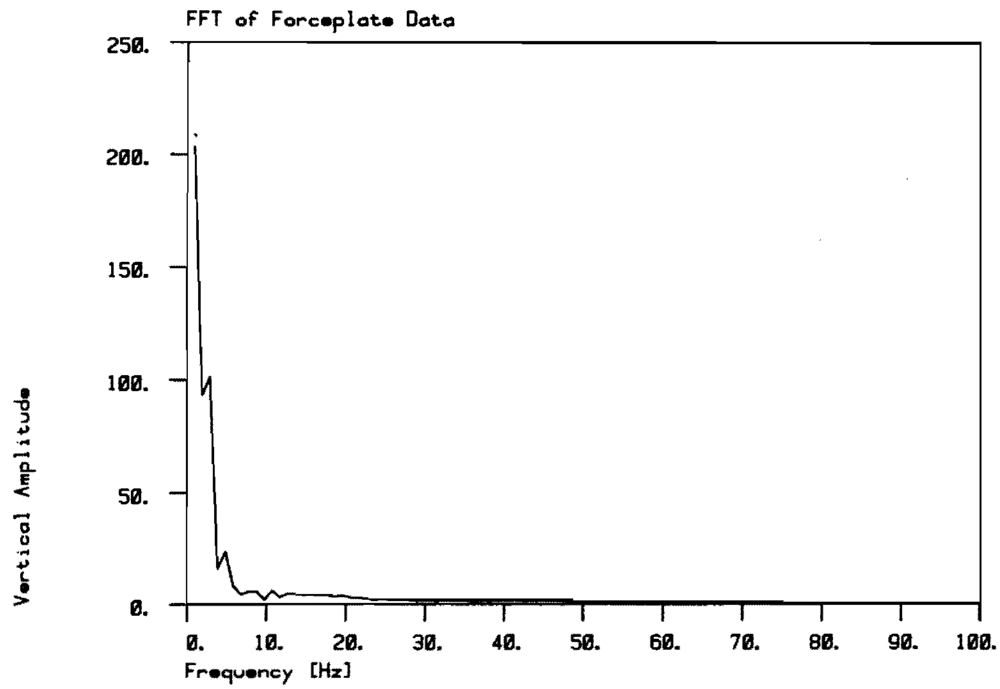

Fig. 9. Frequency domain plot of normal gait vertical force.

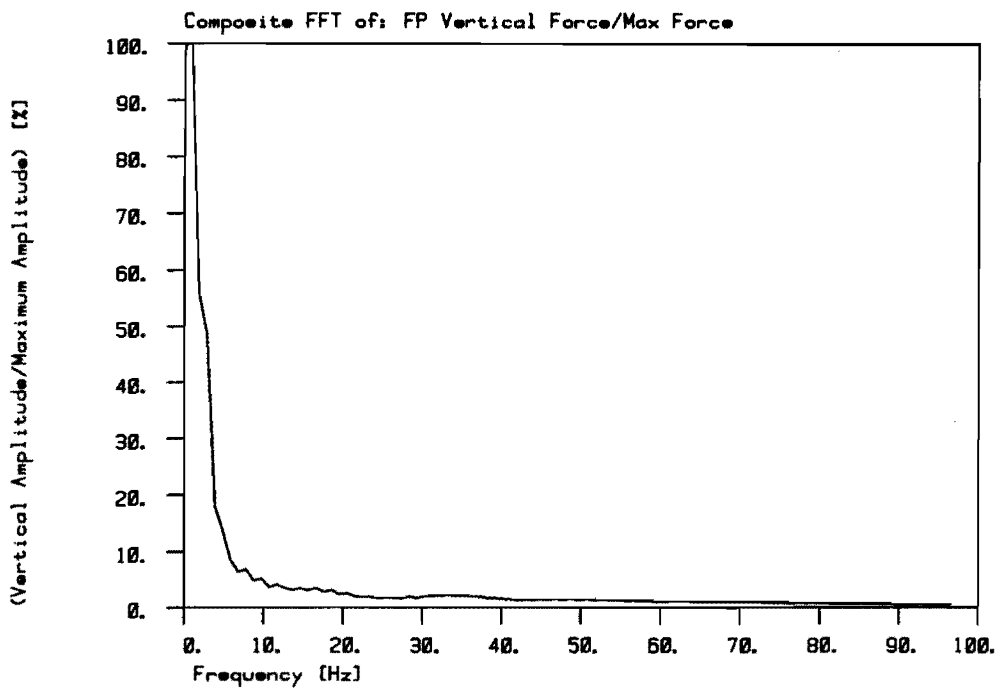

Fig. 10. Composite frequency envelope of foot force records.

rejection and signal corruption. Alternatively these results can be used as a guide in specifying a gait analysis system.

To measure human motion, or to estimate gait dynamics, the *kinematic* measurement system used must be able to produce accurate measures up to the desired frequency. In order to preserve 99% of the signal power in gait, positional fidelity must be maintained up to 15 Hz, which for a sampled system requires a *minimum* of 30 Hz sampling rate. At 30 Hz,

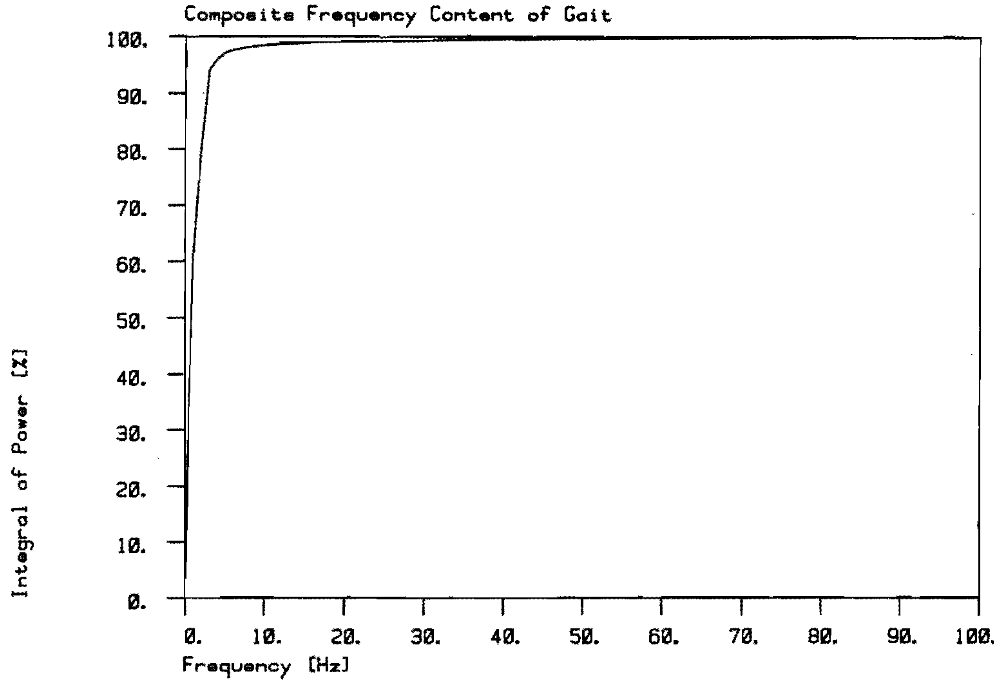

Fig. 11. Integral of power in the composite envelope of foot force records.

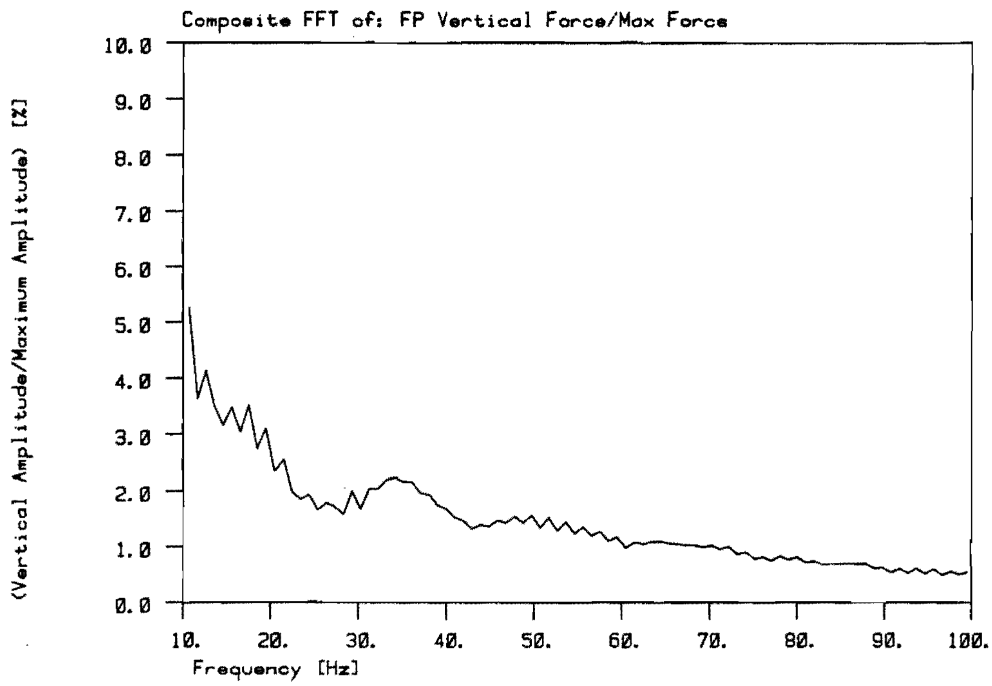

Fig. 12. Expanded section of composite frequency envelope (Fig. 10) between 10 and 100 Hz.

however, the 15 Hz component would only have two data points per cycle, and would be a crude estimate. To properly eliminate effects of noise without aliasing, the sampling rate must be above twice the highest significant noise frequency. A 300 Hz sampling rate will permit measurement of twenty data points per cycle of the highest frequency component of the signal and allow proper noise rejection up to 150 Hz. However, even this high sampling rate, coupled with a low-pass filter will not guarantee only 1% error in position, nor will it guarantee any level of accuracy in the twice differentiated results. The level of accuracy remaining depends on many factors, including the sampled noise (and its derivatives) remaining below the filter cut-off frequency.

The entire mobility measurement and analysis system (including the forceplatform and other opto-electronic equipment, described elsewhere [Antonsson, 1982; Antonsson and Mann, 1983, 1979; Mann and Antonsson, 1983; Mann *et al.*, 1982]) achieves error contributions of no more than 5% in the twice differentiated position data due to noise. The photo-grammetric accuracy of the kinematic measurement system is much higher, in fact only 0.1% errors in position. This indicates that a very highly accurate system is necessary, but not sufficient for accurate gait analysis. A system with small contributions of noise in the range of valid signal frequencies is mandatory.

The importance of measuring and understanding the frequency domain characteristics of any measurement system cannot be overstated. When dynamic estimates are to be produced from double differentiated position data, frequency analysis is vital to producing valid, coherent and fully described results.

Acknowledgements—This research was performed in the Eric P. and Evelyn E. Newman Laboratory for Biomechanics and Human Rehabilitation and funded by: the Department of Mechanical Engineering at the Massachusetts Institute of Technology; the National Institutes of Health, Grant Number R01-AM-16116; the Whitaker Professorship of Biomedical Engineering; the Department of Education, National Institute of Handicapped Research, Grant Numbers: G00-80-03004 and G00-82-00048; and a National Institute of General Medical Sciences Fellowship, Grant Number GM2136.

REFERENCES

Alexander, R. M. and Jayes, A. S. (1980) Fourier analysis of forces exerted in walking and running. *J. Biomechanics* **13**, 383–390.

Antonsson, E. K. (1982) A three-dimensional kinematic acquisition and intersegmental dynamic analysis system for human motion, Ph.D. Thesis, M. I. T., Cambridge, MA.

Antonsson, E. K. and Mann, R. W. (1983) A three-dimensional kinematic acquisition and intersegmental dynamic analysis system for human motion. *ASME Biomechanics Symposium*, University of Houston, Houston, Texas, June 19–22, 1983.

Antonsson, E. K. and Mann, R. W. (1979) Automatic 3-D gait analysis using a selspot centered system. 1979 *Advances in Bioengineering* p. 51. American Society of Mechanical Engineers, New York.

Cappozzo, A., Figura, F., Marchetti, M. and Pedotti, A. (1976) The interplay of muscular and external forces in human ambulation. *J. Biomechanics* **9**, 35–43.

Cappozzo, A., Leo, T. and Pedotti, A. (1975) A general computing method for the analysis of human locomotion. *J. Biomechanics* **8**, 307–320.

Cappozzo, A., Maini, M., Marchetti, M. and Pedotti, A. (1974) Analysis by hybrid computer of ground reactions in walking. *Biomechanics IV* (Edited by Nelson, R. C. and Morehouse, C. A.), pp. 496–501.

Hatze, H. (1981) The use of optimally regularized fourier series for estimating higher-order derivatives of noisy biomechanical data. *J. Biomechanics* **14**, 13–18.

Lanshammar, H. (1982) On practical evaluation of differentiation techniques for human gait analysis. *J. Biomechanics* **15**, 99–105.

Lanshammar, H. (1982) On the precision limits for derivatives numerically calculated from noisy data. *J. Biomechanics* **15**, 459–470.

Light, L. H., McLellan, G. E. and Klenerman, L. (1980) Skeletal transients on heel strike in normal walking with different footwear. *J. Biomechanics* **13**, 477–480.

Mann, R. W. and Antonsson, E. K. (1983) Gait analysis—precise, rapid, automatic, 3-D position and orientation kinematics and dynamics. *Bull. Hosp. Jt Dis.* **43**, 137–146.

Mann, R. W., Rowell, D., Dalrymple, G., Conati, F., Tetewsky, A., Ottenheimer, D. and Antonsson E. (1982) Precise, rapid, automatic 3-D position and orientation tracking of multiple moving bodies. *Biomechanics VIII*, Human Kinetics Publishers, Champaign, IL.

Mizrahi, J. and Suzak, Z. (1982) *In-vivo* elastic and damping response of the human leg to impact forces. *J. biomech. Engng.* **104**, 63–66.

Perry, J., Moynes, D. and Antonelli, D. (1984) Sampling rate for motion analysis. *Transactions of the 30th Annual Meeting of the Orthopaedic Research Society*, Vol. 9, p. 155. ORS, Chicago.

Pezzack, J. C., Norman, R. W. and Winter, D. A. (1977) An assessment of derivative determining techniques used for motion analysis. *J. Biomechanics* **10**, 377–382.

Simon, S. R., Paul, I. L., Mansour, J., Munro, M., Abernethy, P. J. and Radin, E. L. (1981) Peak dynamic force in human gait. *J. Biomechanics* **14**, 817–822.

Soudan, K. and Dierckx, P. (1979) Calculation of derivatives and fourier coefficients of human motion data, while using spline functions. *J. Biomechanics* **12**, 21–26.

Soudan, K., Van Audekercke, R. and Martens, M. (1979) Methods, difficulties and inaccuracies in the study of human joint kinematics and pathokinematics by the instant axis concept. example: the knee joint. *J. Biomechanics* **12**, 27–33.

Stearns, S. D. (1975) *Digital Signal Analysis*. Hayden Book Company, Rochelle Park, NJ.

Voloshin, A., Wosk, J. and Brull, M. (1981) Force wave transmission through the human locomotor system. *J. Biomech. Engng* **103**, 48–50.

Wells, R. P. and Winter, D. A. (1980) Assessment of signal and noise in the kinematics of normal, pathological and sporting gaits. *Proceedings of the Special Conference of the Canadian Society for Biomechanics*, London, Ontario, Canada, 27–29 October, pp. 92–93.

Winter, D. A., Quanbury, A. O., Hobson, D. A., Sidwall, H. G., Reimer, G., Trenholm, B. G., Steinke, T. and Shlosser, H. (1974) Kinematics of normal locomotion—a statistical study based on television data. *J. Biomechanics* **7**, 479–486.

Winter, D. A., Sidwall, H. G. and Hobson, D. A., Measurement and reduction of noise in kinematics and locomotion. *J. Biomechanics* **7**, 157–159.