Trade-Off Strategies in Engineering Design*

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Abstract

A formal method to allow designers to explicitly make trade-off decisions is presented. The methodology can be used when an engineer wishes to rate the design by the weakest aspect, or by cooperatively considering the overall performance, or a combination of these *strategies*. The design problem is formulated with preference rankings, similar to a utility theory or fuzzy sets approach. This approach separates the design *trade-off strategy* from the performance expressions. The details of the mathematical formulation are presented and discussed, along with two design examples: one from the preliminary design domain, and one from the parameter design domain.

1 Introduction

For a robust automation, design decision making methods need to be advanced to represent and manipulate a design's different concerns and uncertainties. This development is crucial, since the preliminary decision making process of any design cycle has the greatest effect on overall cost [3, 6, 14]. In a design decision making process, engineers must trade-off widely differing concepts to realize a result which maximizes their *overall* preference for a design. These concepts are usually incommensurate: for example, they could be as different as cost, degree of safety, degree of manufacturability, or amount of various performance indicators: stress, heat dissipation, etc. This paper presents *design metrics* to represent and manipulate these concerns. These metrics take the form of formal design strategies to permit the designer to trade-off one (or more) parameter(s) against others, and to implement an overall approach to design trade-offs: either conservative, aggressive, or a combination of the two. The terms: conservative and aggressive are defined in the next section. A formal mathematics will be presented to allow designers to explicitly make such trade-off decisions.

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This approach can help designers observe, justify, and direct their decision making processes. In any design scenario, there are multiple goals which need to be achieved [32]. Designers restrict and choose parameter values based on a combination of these concerns. This work will permit designers to directly specify a design goal *trade-off strategy* to specify how to trade-off different design goals, and thus allow observation, justification, and recording of decisions made.

1.1 Design Trade-Off Strategies

Design trade-off strategies are always present in the design process. For example, in the design of a spacecraft solar power cell, a strategy might be to trade-off the performance gains of some goals (like available power output) to increase the level of other aspects deemed marginal (like stress), to ensure the cell will always function. We will use the term "conservative" design strategy, or also "non-cooperating" or "non-compensatory" strategy, to describe a design strategy of trading off to improve the lower performing goals. A design's overall preference will be based on the attribute with the lowest preference. Other attributes with higher preference do not compensate for the attribute(s) with lower preference.

On the other hand, a designer may wish to slightly reduce some of the weaker goals in a design if large gains can be made in the other goals, which would more than compensate for the slight loss. For example, in the design of a sports car, the designer might reduce the safety margin of some variables (like stress) to gain in performance of other variables (like horsepower), even though the stress may already be quite high. We will use the term "aggressive" design strategy, or "cooperating" or "compensatory" strategy, to describe a design strategy of always cooperatively trading off the goals to improve the design. Obviously, hybrid forms of these approaches exist and are used, where some portions of a device are designed conservatively, and other portions aggressively. We will use the terms: conservative and aggressive design strategies throughout this paper.

1.2 Designer Preferences

A method for representing and manipulating uncertainties in preliminary design, to formalize the process of making these trade-off decisions, has been introduced and developed by Wood and Antonsson, [34, 35, 36], called the *method of imprecision*. It is used to compare and contrast different objectives within a proposed design and among different design alternatives. The intent is to determine proposed candidates' feasibility and limitations, even with uncertainty in the variables used.

This paper will incorporate the concepts of imprecision, and a brief review will be presented here. Imprecision indicates a designer's uncertainty in selecting a value for a parameter, in the form of a zero to one rank. If a designer prefers to use a value for a parameter, it will be ranked high, near one. On the other hand, if a designer does not prefer a parameter's value, it will be ranked low, near zero. Depending on the domain, the preferences are specified on actual physical variables (such as model dimensions, or calculable quantities such as stress), or, in the preliminary design domain (for example), on features in the design. The parameters (on which the preferences are placed) depend on the domain of the design. The discussion presented in the paper will be appropriate for any stage of the design process, from the early planning stages through production. Examples from both the preliminary and a latter stage of the design process will be presented. In all cases, the scheme is to place preferences on the candidate model features, with the aim of determining an overall preference for each candidate model to determine which to pursue. The reader is referred to [20, 21, 22, 33, 34, 35, 36] for a discussion on how to specify preferences; this paper will not discuss this aspect of the problem. Rather, this paper will focus on the task of combining these individual preferences (of the different parameters' values) to obtain a preference rank for the vector of design parameter values.

The method of imprecision as developed to date used the mathematics of fuzzy sets to perform this combination [34]. The primary objective of this paper is to introduce different methods for combining these preferences, and to show that these different methods effectively represent different design strategies the designer may adopt.

1.3 Related Work

There has been some progress in the development of optimization methods with preference functions. Diaz [7, 8, 9], Rao [20, 21, 22], and Sakawa and Yano [24, 25, 26] have advanced the use of "fuzzy goals" where the objective functions and constraints consist of fuzzy preference functions on different performance parameters. This paper will illustrate the implications on the choice of the form of the fuzzy mathematics used. That is, if conventional fuzzy mathematics is used, it will be demonstrated that this combination corresponds to using a conservative design strategy.

Parallel work by Dubois and Prade considers trading-off multiple goals with preference functions [11]. In [11], they review connectives which could be used to combine goals. However, they provide no compelling reason for selecting any of the possible candidates (as is done here: by specifying a design trade-off strategy). Also, the candidates they propose for combining preferences on incommensurate goals have too many restrictions for the purposes of engineering design. Their developments are derived from the realm of uncertain logic. As such, they are primarily interested in uncertain versions of conjunction and disjunction. In classical logic, these operations are commutative and associative, which Dubois and Prade assume for all of their developments. In general, engineering design connectives should not be commutative. It makes no sense, for example, to require one goal's weighting to be applicable to a different goal, which is as commutativity requires. Yager, in [37], introduces some of the mathematics that we present here, in the context of selecting from a finite set of alternatives. The relationship of his developments to ours will be discussed below.

An alternative to the use of imprecision is utility theory [19, 29]. Utility theory trades off goals by specifying utility curves on each goal, and then maximizes overall utility by aggressively combining the goals. Doing so eliminates the conservative design strategy from consideration, which could be the design strategy of choice in some cases.

In domains involving goals with explicit expressions, one could formulate the design problem using an optimization methodology [16]. Such single objective formulations have been argued to be constraining for actual design problems [32]. Instead, multi-objective function formulations could be used [12, 28, 32]. The methodology presented here is compatible with these multi-objective function algorithms, in that one can use them to solve the formulations presented here, when the domain has sufficient formalization (performance parameter equations) [28]. The focus of this paper is on formally specifying the multi-criteria objective function, not methods for finding its global peak.

In the preliminary design domain, the degree of specification of candidate models is usually incomplete. The method of imprecision can still be used, however, to determine which candidate models offer the most promise. Traditional methods used in this stage of design are matrix methods [2]. Current advanced versions are QFD [1, 13] and Pugh's method [17]. The basis for the combination procedure of such matrix methods will be discussed in an example below.

2 Design Imprecision

In the method of imprecision, designer preferences (μ) are represented on a scale from zero to one, with preferences placed individually on each parameter. We shall denote *design parameters* (DP) as those parameters whose values are to be determined as the objective of the current design process, *e.g.*, lengths, materials, etc. We shall denote *performance parameters* (PP) as those parameters whose values depend on the design parameter values, and which give indications of performance, *e.g.*, stress, horsepower, etc. This paper presents a method to determine an overall rank for a *vector* of design parameters ($\overrightarrow{DP}$) given the individual preference information. The preference information is specified both on the design parameters and on the various performance parameters in the form of specifications or requirements.

Section 2.1 will present axioms governing any preference combination metric. Sections 2.2 through 2.5 will discuss different functions to use as global design metrics, and their relation to design strategies. The discussion will be in the context of parametric design; however, the developments apply to any design stage, as the examples will demonstrate.

2.1 Global Preferences

The objective of this paper is to formalize an approach to define and identify a "best" engineering design. However, the various parameters in the design usually reflect incommensurate concepts, and therefore should be combined using a construct that they share: designer preference. This means that preference information on the design parameters (DPs) and requirement preferences on the the performance parameters (PPs) must be combined into an overall preference rating ($\mathcal{P}$) for that design parameter set ($\overrightarrow{DP}$). Therefore, to combine designer preferences, a global design connective, or metric, is defined, expressed as a function of the known preferences of the goals:

$$\mu(\overrightarrow{DP}) = \mathcal{P}\left[\mu(DP_1), ..., \mu(DP_n), \mu(PP_1(\overrightarrow{DP})), ..., \mu(PP_q(\overrightarrow{DP}))\right] \tag{1}$$

This statement implies that the choice of a design parameter set is based on combining (in a yet to be determined fashion) the preferences of the design parameters and performance parameters. It is a formalization of the idea that designers combine incommensurate parameters based on how much each parameter satisfies them.

The design problem is then to find the design parameter set which maximizes the overall preference:[1]

$$\mu(\overrightarrow{DP}^*) = \max_{DPS}\left[\mathcal{P}\left[\mu(DP_1), ..., \mu(DP_n), \mu(PP_1(\overrightarrow{DP})), ..., \mu(PP_q(\overrightarrow{DP}))\right]\right] \qquad (2)$$

The most preferred design parameter set $\overrightarrow{DP}^*$ is the one which maximizes $\mathcal{P}$ across the design parameter space (DPS), which is the set of all design parameter combinations. This is a formalization of the idea that designers choose the design parameter set which maximizes their overall preference.

The choice of method to combine preferences ($\mathcal{P}$) is determined by the design strategy. For this reason, the minimum function ($\mathcal{P} = min$) applied to all of the preferences is not automatically acceptable, as pointed out in [36] (the *min* is commonly used in fuzzy mathematics to combine information). This development shall discuss when different functions are appropriate to use as $\mathcal{P}$. First, a set of axioms with which all proposed resolving functions (to use as connectives, or metrics) must be consistent (at least those which operate with preferences) is introduced in Table 1. Then example functions will be given, and it will be shown when each is appropriate for different problems.

The first axiom in Table 1 is a boundary condition requirement. It states that if the designer prefers absolutely all of the goals (preference $\mu = 1$), then the design will also be preferred absolutely. Similarly, if the designer has no preference for the value of any one of the goals (preference $\mu = 0$), then the overall design (as a set of goals) will also not be preferred. Weighted sum multi-criteria objective formulations do not conform to this axiom, as discussed by Biegel and Pecht in [5]. Vincent [32] also presents this argument in the case of (non-preference) multi-objective function optimization.

The second axiom is a monotonicity requirement. It states that if an individual goal's preference is raised or lowered, then the design's overall preference is raised and lowered in the same direction, if it changes at all. Hence, in a multi-component design, if one component's preference is increased with the other components' preference remaining the same, then the design's overall preference does not go down. The axiom does not mean the preferences or the performance parameters must be monotonic. If either the preferences specified or the performance parameters used are non-monotonic, then this axiom ensures that $\mathcal{P}$ will monotonically propagate the non-monotonicities.

The third axiom is a continuity requirement. It states that as an individual goal's preference is changed slightly, then the overall preference for the design will change at most slightly. It does not mean the preference for any goal must be continuous. It states only that as any individual goal's preference is continuously changed, the method of combining all the goals' preferences (that is, $\mathcal{P}$) will induce only continuous changes in the overall preference, if it changes at all. If some parameters have preference discontinuities, the method of combining them will continuously propagate the discontinuities. Therefore a design will not be abruptly preferred by slight changes in values, unless the parameterizing expressions dictate this.

These first three axioms present nothing new in terms of inferencing mechanisms under uncertainty. Probability and Bayesian inferencing [31], Dempster-Shafer theory [27], fuzzy sets and triangular norms in general [10], and finally utility theory [19] all conform to these

[1] Throughout the paper *max* is used to to mean *sup* or *least upper bound*, and *min* is used to mean *inf* or *greatest lower bound*.

axioms, with slight variations on boundary conditions. The subsequent discussion, however, indicates where these theories diverge among themselves and with the development presented here.

The last axiom in Table 1 is an idempotentcy restriction. It states that if a designer has the same preference for all individual concerns in a design, then the overall preference must have this degree of preference as well. This condition must be considered closely, for it is a statement related to rationality. A definition of irrational behavior is to act in a manner which is against one's objectives [31]. In the context of preference, this linguistic definition translates into meaning that an irrational method is to reduce or increase the overall preference beyond what the parameters specify. Formalization of this definition, however, has many possibilities (among which, for example, is probability, or even $\mathcal{P}$ as so far specified) since this definition is linguistic and non-formal.

The last axiom of idempotentcy eliminates any functions which combine preferences in an inherently pessimistic or optimistic manner. Methods which combine individual preferences and artificially reduce or increase the overall preference rank should not be considered: *e.g.*, some of the various triangular norms [10] and power methods [8]. For example, if a design had two goals, each with preference 0.8, one would not expect an overall rating of 0.2, or 1.0, since these results are irrational: they reduced or increased the overall preference beyond what the parameters specified. Idempotentcy eliminates these possibilities.

One axiom absent from the list is strictness:

$$\forall j, \mathcal{P}(\mu_1, ..., \mu_j, ...) < \mathcal{P}(\mu_1, ..., \mu_j', ...) \text{ iff } \mu_j < \mu_j' \tag{3}$$

The strictness requirement is unacceptable as *always* being required for *any* design metric. For some design strategies, strictness may be acceptable; for others not. For example, consider one parameter in two different designs which has a low preference of 0.4. The two designs differ only in that a second parameter (different from the one ranked at 0.4) has a preference of 0.6 and 0.8 in the two designs respectively. See Figure 1. It is not clear that the designer should *always* distinguish between these two designs, which the strictness requirement requires. Both designs have equally bad components at 0.4, and so the designer may decide to rank both designs as equally poor overall, with a preference of 0.4. Alternatively, the designer may see the first parameter as irrelevant, and rank the second design as better overall. This decision depends on the strategy employed. However, attempting to *always* include the strictness requirement eliminates valid strategies from consideration by always differentiating between Design 1 and 2 in Figure 1. The strictness requirement is discussed in [10].

Table 1 is a list of necessary requirements to which any global combination design metric which uses preferences must conform. Using fewer constraints on the design metric permits irrational and non-intuitive preference combination functions to be used, based on the informal comparison of these axioms with design decision making. Additional constraints will now be placed on the metric, where these additional constraints imply a particular design strategy.

2.2 Conservative Design

Suppose the designer wishes to trade off to improve the lower performing goals (in terms of preference) when selecting a design parameter set $\overrightarrow{DP}$. Also, assume for the moment that all of the individual goals are equal causes of concern to the designer. This implies that, to improve a

Table 1: Overall Preference Resolution Axioms.

$\mathcal{P}(0, \mu_{j_1}, \ldots, \mu_{j_{n+q-1}}) = 0 \qquad \mathcal{P}(1, ..., 1) = 1$	(boundary conditions)
$\forall j, \mathcal{P}(\mu_1, ..., \mu_j, ...) \leq \mathcal{P}(\mu_1, ..., \mu'_j, ...)$ iff $\mu_j \leq \mu'_j$	(monotonicity)
$\forall j, \mathcal{P}(\mu_1, ..., \mu_j, ...) = \lim_{\mu'_j \to \mu_j} \mathcal{P}(\mu_1, ..., \mu'_j, ...)$	(continuity)
$\mathcal{P}(\mu, ..., \mu) = \mu$	(idempotentcy)

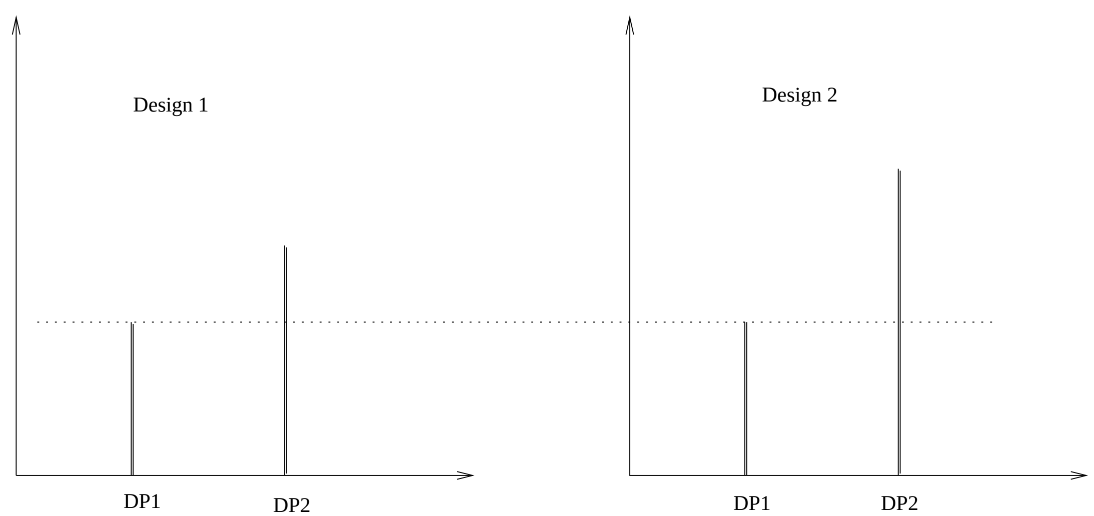

Figure 1: Two designs which have the same low preference for a component DP_1, and different (higher) preference for a different component DP_2.

design, there must be an increase in the preference level of the goal whose preference is lowest. We refer to this as a conservative design strategy, and the method to use for combining the multiple preferences is $\mathcal{P} = min$. That is,

$$\mu(\overrightarrow{DP}^*) = \max_{DPS} [\min[\mu(DP_1), ..., \mu(DP_n), \mu(PP_1), ..., \mu(PP_q)]] \tag{4}$$

where $\overrightarrow{DP}^*$ is the most preferred design parameter solution set. Using the *min* as a design metric always improves a design's worst aspect, meaning that aspect with the lowest preference. Whichever parameter has the lowest preference dictates the overall preference. If the designer can improve the design, this parameter will change. Of course, the goal which is the "weakest link" changes with changes in $\overrightarrow{DP}$ (changes of position in the design space). Therefore this metric trades off to improve the lower performing goals.

Finally, Equation 4 is exactly the fuzzy set formulation of the design problem [8, 20]. Therefore, using a fuzzy set resolution in the design domain reflects trading off goals conservatively, and without considering importance weightings.

2.3 Aggressive Design

The *min* resolution of Equation 4 is not always appropriate, however. If the resulting design is drastically hindered by one parameter and relaxing it a bit greatly increases the others' preference, then the modified design may be considered to produce a higher "overall" performance, even though the lower performing goal was slightly reduced even further. In this case, the hindering parameter should be relaxed and thereby allow other parameters to substantially increase their preference.

This can be accomplished with the use of a product:

$$\mu(\overrightarrow{DP}^*) = \max_{DPS} \left[\prod_{j=1}^{q+n} \mu_j \right]^{\frac{1}{q+n}} \tag{5}$$

where n is the number of design parameters and q is the number of performance parameters.

This resolution reflects a different design strategy than the *min* resolution presented earlier. Specifically, Equation 5 allows higher performing goals to compensate for lower performing goals (in terms of preference). This metric trades off the goals to cooperatively improve the design. We refer to this as an aggressive (or cooperative) trade-off strategy.

2.4 Importance Ratings

Both strategy formalizations presented in the previous two sections assumed all goals were equally important. The formalization of the conservative design strategy as reflected by Equation 4 traded off the overall performance to gain in the lower performing goals, as if each were equally important to the designer. The formalization of the aggressive design strategy as reflected by Equation 5 did the reverse (traded off the lower performing goals to gain in overall performance), as if each goal were equally important. Yet, in the general case, each goal will not hold equal importance. In this more general case, factors must be included to allow the designer to specify how much concern (or weight) should be allocated to each goal.

The reader is referred to [23, 28] for methods on how to specify weights; this paper will not discuss this aspect of the problem. However, it is noted that there are several reasons why weighting functions present difficulty [28, 30]. We concur, and adopt the standard solution to the problem of specifying weights: iteration. That is, it is not assumed the designer can, *a priori*, specify the final goal weights, only preliminary estimates. The designer then gains insight on how to specify weights through iteration. In any case, techniques for specifying weights from pairwise comparisons of goals are the Analytical Hierarchy Process [23], or the marginal rate of substitution [28]. It is noted that, though theoretical issues remain with weighting functions, they are commonly used in practice [1, 2, 13, 18].

Assigning importance factors to goals is a relative measure: a goal's importance is ranked relative to the rest of the goals in a design. The importance of goal j (either a design parameter or a performance parameter) shall be denoted ω_j. Since importance is a relative measure, the importance factors should always be normalized by their sum; *i.e.*, the ω_j must be such that

$$\sum_{j=1}^{q+n} \omega_j(\vec{p}) = 1 \tag{6}$$

where $\vec{p}$ is the vector composed of the design and performance parameters. This allows for non-normalized weights; for example, ω_j might be fuzzy. At each point, the non-normal weights must be normalized.

There is another observation on the importance factors: since it is assumed that no goals are trivial or absolutely dominant, the normalized ω_j must be such that

$$0 < \omega_j(\vec{p}) < 1 \qquad \text{for all } j \tag{7}$$

The 0 lower boundary condition is actually not strict: the particular goal j then simply drops out of the consideration ($\mu_j^{\omega_j}$ becomes 1). Further, the 1 upper boundary condition is always ensured by the previous normalization requirement.

A final observation is that importance factors are functions: they can change with changes in the design. If a goal's preference is low, perhaps a designer may wish to change the goal's importance. It is assumed that slight changes in a goal's value do not induce drastic changes in the goal's importance. This is a continuity requirement; *i.e.*, the normalized ω_j must be such that

$$\lim_{\vec{p}' \to \vec{p}} \omega_j(\vec{p}') = \omega_j(\vec{p}) \tag{8}$$

Having made these observations about importance factors, they can now be used in any design strategy. For the conservative design strategy, the design metric becomes:

$$\mu(\overrightarrow{DP}^*) = \max_{DPS} \left[\left(\min_{i \in [1,q+n]} [\mu_i^{\omega_i}] \right)^{\frac{1}{\max_{i \in [1,q+n]} [\omega_i]}} \right] \tag{9}$$

This expression reflects trading off the overall performance to gain in the lowest performing goal, with each goal raised to its importance level. In the previous unweighted case (Equation 4), each goal had an equal importance of $\frac{1}{q+n}$. Equation 9 reduces to Equation 4 when all goals have equal importance ($\omega_j = \frac{1}{q+n}$ for all j).

An almost identical function has been proposed by Yager [37] for including weighting functions into fuzzy sets. However, our metric is normalized to maintain consistency with Table 1. Therefore it is a normalized metric, enabling direct preferential comparisons with other alternatives for which the designer may not have used the conservative design strategy. A different technique was proposed by Bellman and Zadeh [4] involving the fuzzy linear weighting of goals. Their formalization is not adopted because of its failure to maintain consistency with Table 1. They fail to maintain consistency with the boundary conditions. Hence one could select a design parameter set which has no preference for a subset of the goals. As stated, Vincent [32] and Biegel and Pecht [5] also argue this is unacceptable for engineering design. Dubois and Prade extend Bellman and Zadeh's technique into possibility theory [11], where the weights become degrees of possibility. The combination of possibility with preference is an area for future research.

The conservative design strategy is affected by the use of importance factors (ω_j) as will be demonstrated graphically for a simple case. Consider a design with just one parameter which has preferences from two sources, as shown in Figure 2. For example, the parameter might be material ultimate strength, μ_1 might be preference for cost (cheaper materials are more preferred), and μ_2 might be preference for strength (stronger materials are preferred more). As the relative importance of the preferences change (ω_1 goes from an importance of 1.0 to an importance of 0.0 as ω_2 goes from 0.0 to 1.0), the resulting peak preference point changes as shown in Figure 3. The design strategy will choose the value with maximum preference from the resulting combination. For example, with $\omega_1 = 0.75$ and $\omega_2 = 0.25$, the final parameter value chosen will equal 0.44, with a preference of 0.75 (the boxed point in Figure 3).

For the aggressive (cooperating) design strategy case, the design metric will use a variation from the previous unweighted case (Equation 5):

$$\mu(\overrightarrow{DP}^*) = \max_{DPS}\left[\prod_{i=1}^{q+n} \mu_i^{\omega_i}\right] \tag{10}$$

This expression reflects trading off the goals cooperatively to gain in the overall performance, with each goal raised to its importance level. In the previous unweighted case (Equation 5), each goal had an equal importance of $\frac{1}{q+n}$. Equation 10 reduces to Equation 5 when all goals have equal importance ($\omega_j = \frac{1}{q+n}$ for all j).

Yager presents this resolution in [37] as a method to select a proper course of action based on a set of objectives. We, however, present a justification for its use as reflecting a design trade-off strategy, and apply the method to problems beyond selection from a finite set of alternatives, the thrust of Yager's work.

The aggressive design strategy is also affected by the use of importance factors (ω_j) as will be demonstrated graphically for the same simple example (Figure 2). As the relative importance of the preferences change (ω_1 goes from an importance of 1.0 to an importance of 0.0 as ω_2 goes from 0.0 to 1.0), the resulting peak preference point changes as shown in Figure 4. The aggressive design strategy will choose the value with maximum preference from the resulting combination. For example, with $\omega_1 = 0.75$ and $\omega_2 = 0.25$, the final parameter value chosen will equal 0.4, with a preference of 0.78 (the boxed point in Figure 4).

Note that, for the same problem with the same preferences and importance factors, the two design strategies selected different peak points. The two strategies traded off the goals in

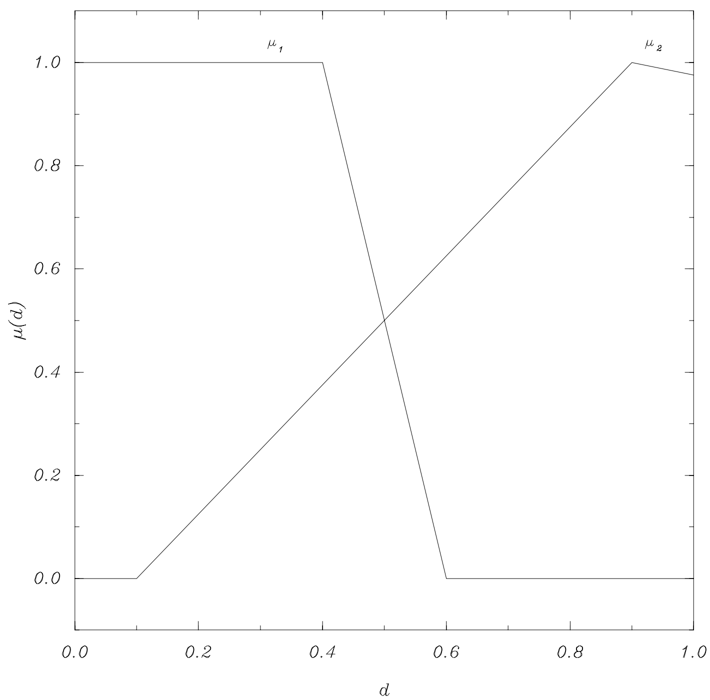

Figure 2: Single parameter design with two preference sources.

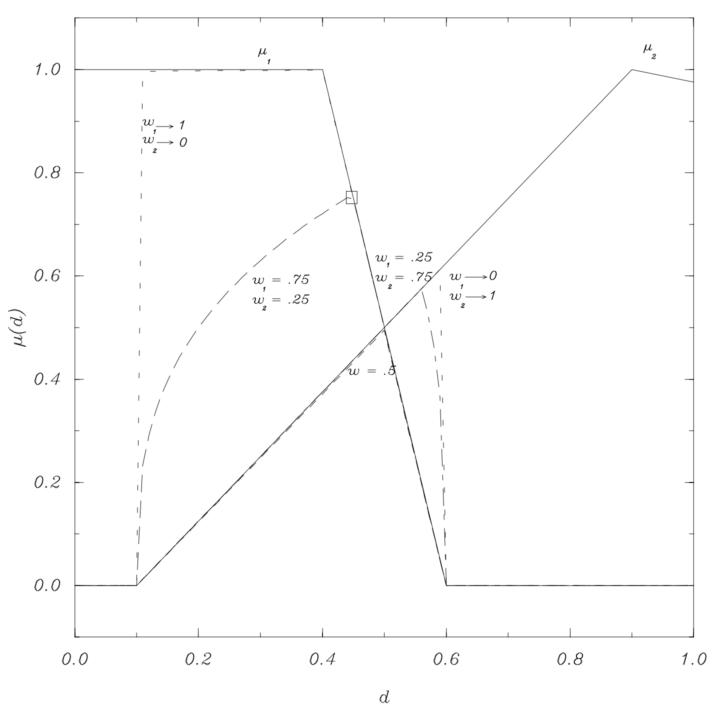

Figure 3: Weighted conservative design strategy results.

different fashions: conservatively or aggressively. In either case, a goal's importance can be handled within the design strategies.

2.5 Hybrid Design Strategies

Generally, a designer may not wish to exclusively trade-off every design component aggressively or conservatively. A subsystem may need to have its weakest goals maximized, but a different subsystem may need to be cooperatively maximized. For these more general cases, a combination of the two methods (the *min* and the *product*) can be performed, and this is consistent with Table 1's axioms. The sub-design would use the *min* combination of its preference rankings, and this sub-result would use the *product* to be combined with rest of the design.

In the general case, an entire hierarchy of the parameters' preferences would be constructed into the overall metric. This construction could be aided by using the γ-*level* measure [34] to determine which parameters are critical to the design. The γ-*level* measure provides an indication of how sensitive each design parameter is to each performance parameter, based on their relation and the specified preferences. If the γ-*level* measure indicates a particular design parameter is critical to different performance parameters, the designer could then take extra care when specifying that design parameter's importance. Also note that the importance weightings might also change as the design process progresses to reflect the addition of more information.

2.6 Discussion: What is a Design Strategy?

The term "strategy" has many meanings, both in the research literature, and in engineering practice. This paper has introduced a formalization of one aspect of design strategies: how to make trade off decisions among different goals. Design strategies might also include considerations of performance, safety, importance, or noise variations. They also usually include considerations of the design problem solving methodology or approach. This paper's use of the term "strategy" therefore includes only one aspect: how to make trade-off decisions among multiple, incommensurate goals in a design.

A related question is how to determine what goals should be included in a problem's formalization. This question cannot be answered *a priori*, but will evolve with the design. A preliminary indication of whether a parameter needs consideration can be determined in the same manner as developed for utility theory: using Ellis' "test of importance" [19]. In this method, before determining how the overall metric is to be formulated, the designer asks whether a parameter's inclusion could change the choice of the others. If so, this additional parameter should be included in the formalization. Hence every possibly important parameter in a design is included. This would likely lead to overly complicated forms. Therefore, the γ-*level* measure [34] could be used to eliminate those which are shown to have little consequence.

Another concern involves practically evaluating (or searching for) the most preferred design parameter set, once a strategy has been used to formally specify the multi-criteria objective function. This aspect of the problem will not be elaborated; it is a problem and processor dependent consideration. For example, when the design space is a list of alternative configurations and the goals are features of the design, the problem can be formulated in a matrix format (even for designs involving hundreds of variables) with preferences entered in the matrix. Search is then simply selecting the alternative with maximum $\mathcal{P}$ of the feature preferences, as will be shown in Example 1. In such a case, for a human "processor", the *min* conservative strategy is easier to evaluate than the *product* aggressive strategy.

A different problem may involve goals with explicit performance parameter expressions, as will be shown in Example 2. Here optimization methods [16] can be invoked to search across the design parameter space for the maximum preference point, possibly involving penalty methods [16] to ease the search. Here the *product* aggressive strategy may be easier to evaluate due to differentiability. The *min* conservative strategy becomes a traditional *maximin* multiple goal optimization formulation [15]. In any case, iteration will almost certainly be required to ensure the final preferences and weights. Formal iterative methods (see Steuer [28] for a review) could be used.

3 Examples

The first example presented will be in the preliminary design domain. The task is to determine which of two candidate models to pursue into the latter design stages. The second example will be in the parametric design domain. The task is to determine which of two air tanks to manufacture, and which parametric design parameter values to use.

3.1 Example 1: Preliminary Design

Consider the design task involving a selection between two candidate concepts. The candidates are to be used for assembling items in a manufacturing production line. The first candidate design is a special purpose mechanism, the other is a general purpose robotic arm.

The decision criteria for determining which candidate to pursue into the subsequent design stages are listed in Table 2. As well, each criterion's importance (on a scale of 0 to 5), and each candidate's ability to satisfy the criterion (on a scale from -5 to 5) is tabulated. Background and details of matrix methods are discussed in [2, 17].

Using the standard weighted sum matrix analysis [2], the mechanism candidate produces an overall rank of 54, and the robot candidate produces an overall rank of 43. This technique guides the designer to pursue the mechanism.

Using the method of imprecision, the ranks are normalized by the range of the ranking. As well, the importance ratings are normalized by their sum. The results of this calculation are shown in Table 3.

Strategies for resolving these multiple attributes of the candidates can be invoked. Let us assume that the designer wishes to trade-off the criterion in an aggressive, cooperative fashion, meaning that the designer is willing to measure the overall preference of each alternative based on a composite of its attributes. This implies that some goals with high preference can compensate for others with low preference. Then Equation 10 can be used to combine the preferences. Doing so results in a rating of 0.65 for the mechanism, and 0.63 for the robot. Again, the mechanism is determined to be the most promising candidate to pursue.

Now instead, let us assume that the designer wishes to trade-off the goals in a conservative, non-compensatory fashion, meaning that the designer will measure the overall performance for each alternative based on the worst (lowest preference) attribute. This implies that the attributes that perform well cannot compensate for those that perform poorly. Then Equation 9 can be used to combine the preferences. Doing so results in a rating of 0.40 for the mechanism, and 0.48

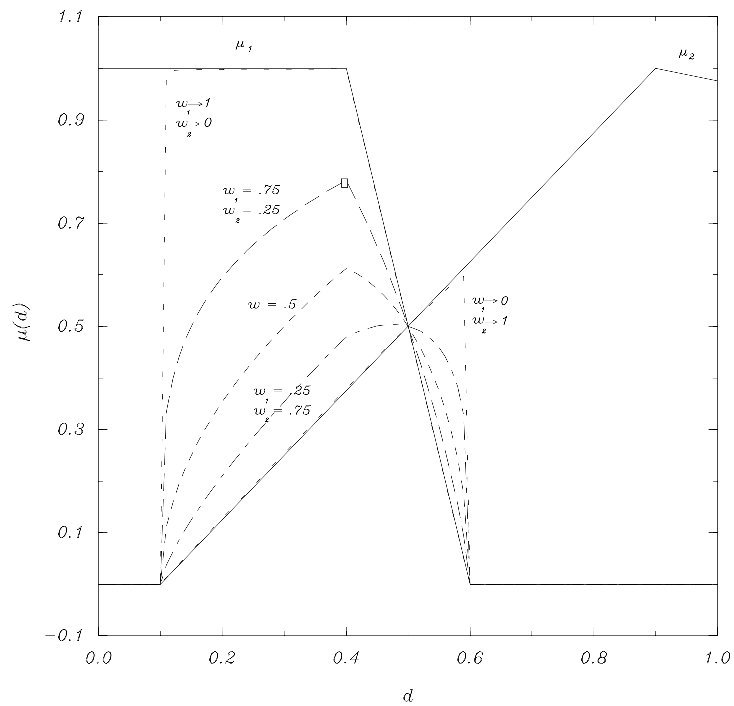

Figure 4: Weighted aggressive design strategy results.

Table 2: Raw designer rankings.

Criteria	Importance	Mechanism	Robot
Ease to get design to satisfy quantity rate	4	5	−1
Ease to ensure operator safety	4	−1	0
Development cost	5	−1	3
Ease to ensure production reliability	5	3	0
Ease to ensure size constraints	2	4	2
Ease to do design by production time	3	0	4
Ease to ensure production quality	4	5	4

for the robotic arm. Here, the robot is determined to be the most promising candidate to pursue. This result is different from the cooperative trade-off strategy result. The new choice was caused by the mechanism being rated poorly at development cost, which was not compensated for by the other superior ratings of the mechanism.

Note the standard matrix method resolved the most promising candidate by selecting the one with the highest weighted performance average across the goals. It did not do so by rating each candidate by the worst aspect. Therefore the standard weighted sum matrix technique invokes a compensating goal trade-off strategy, informally similar to our aggressive trade-off strategy. The developments described here allow for a variety of design strategies. Combinations of conservative and aggressive strategies could be used for different sub-arrangements of the goals, and then these sub-arrangements combined with either a conservative or aggressive strategy, depending on the designer's judgments.

3.2 Example 2: Parametric Design

The example presented below considers a pressurized air tank design, and is the same problem as presented in Papalambros and Wilde [16], page 217. The reader is referred to the reference [16] to see the restrictions applied to the problem to permit it to be solved using crisp constraints and various optimization techniques (monotonicity analysis, non-linear programming). The example is simple and was chosen for that reason, and also the ability of its preferences to be represented on a plane for a visual interpretation.

The design problem is to determine length and radius values in an air tank with two different choices of head configuration: flat or hemispherical. See Figure 5.

There are four performance parameters in the design. The first is the metal volume m:

$$m = 2\pi K_s r^2 l + 2\pi C_h K_h r^3 + \pi K_s^2 r^2 l \tag{11}$$

This parameter is proportional to the cost, and the preference ranks are set because of this concern. Another performance parameter is the tank capacity v:

$$v = \pi r^2 l + \pi K_v r^3 \tag{12}$$

This parameter is an indicator of the design's principle objective: to hold air. This parameter's aspiration level ranks the preference for values. Another parameter is an overall height restriction L_0, which is imprecise:

$$l + 2(K_l + K_h) r \leq L_0 \tag{13}$$

Finally, there is an overall radius restriction R_0, which is also imprecise:

$$(K_s + 1) r \leq R_0 \tag{14}$$

The last two performance parameters have their preference ranks set by spatial constraints.

The coefficients K are from the ASME code for unfired pressure vessels. S is the maximal allowed stress, P is the atmospheric pressure, E is the joint efficiency, and C_h is the head

Table 3: Imprecise designer rankings.

Criteria	Importance	Mechanism	Robot
Ease to get design to satisfy quantity rate	$\frac{4}{27}$	1.0	0.4
Ease to ensure operator safety	$\frac{4}{27}$	0.4	0.5
Development cost	$\frac{5}{27}$	0.4	0.8
Ease to ensure production reliability	$\frac{2}{27}$	0.8	0.5
Ease to ensure size constraints	$\frac{3}{27}$	0.9	0.7
Ease to do design by production time	$\frac{3}{27}$	0.5	0.9
Ease to ensure production quality	$\frac{4}{27}$	1.0	0.9

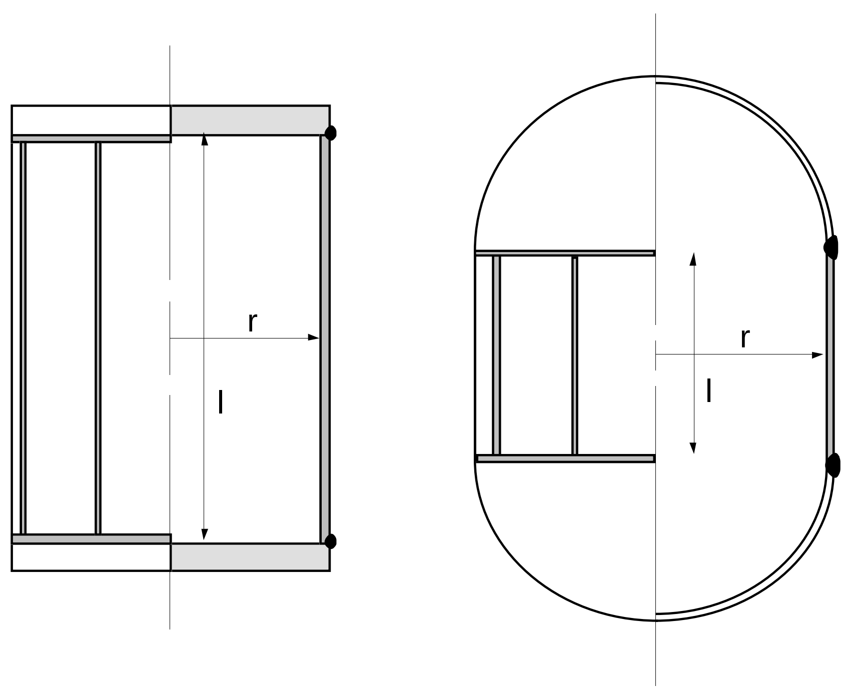

Figure 5: Hemispherical and flat head air tank designs.

volume coefficient.

$$K_h = \begin{cases} 2\sqrt{CP/S} & \text{flat} \\ \frac{P}{2S-.2P} & \text{hemi} \end{cases} \tag{15}$$

$$K_l = \begin{cases} 0 & \text{flat} \\ 4/3 & \text{hemi} \end{cases} \tag{16}$$

$$K_s = \frac{P}{2SE - .6P} \tag{17}$$

$$K_v = \begin{cases} 0 & \text{flat} \\ 1 & \text{hemi} \end{cases} \tag{18}$$

This example's design space is spanned by 2 design parameters l and r. The preferences for values of these design parameters and the four performance parameters are shown in Figures 6 through 11 for the hemispherical design; the flat head design space is similar.

The problem is to find the values for l and r which maximize overall preference. For comparison, both a conservative and an aggressive strategy will be presented and contrasted below. Both consider all goals to be equally important.

For the conservative design strategy, l^* and r^* are to be found, where

$$\mu(l^*, r^*) = \max_{l,r} \left[\min[\mu_l, \mu_r, \mu_{v(l,r)}, \mu_{m(l,r)}, \mu_{L_0(l,r)}, \mu_{R_0(l,r)}] \right] \tag{19}$$

This will find the l^* and r^* by trading off the goals to improve the lowest performing goal (in terms of preference), even though the design parameters and performance parameters are incommensurate with each other.

For the aggressive design strategy, the problem to be solved is to find l^* and r^* where

$$\mu(l^*, r^*) = \max_{l,r} \left[\mu_l \times \mu_r \times \mu_{v(l,r)} \times \mu_{m(l,r)} \times \mu_{L_0(l,r)} \times \mu_{R_0(l,r)} \right]^{1/6} \tag{20}$$

This will find the l^* and r^* by trading off the goals cooperatively among each other, allowing the higher performing goals to compensate for the lower performing goals (in terms of preference), even though the design parameters and performance parameters are incommensurate with each other.

The preference combination results can be seen graphically in Figures 12 through 15. For the conservative design strategy, the *min* of each individual preference across the design space is the resulting surface shown. This is shown in Figures 12 and 13. The surface's maximum value in μ is the solution point to use (the most preferred l and r). For the aggressive design strategy, the individual preference surfaces are multiplied together as a product of powers for all points on the l, r plane. This is shown in Figures 14 and 15. These overall preference surfaces should be compared with the individual goals' preferences shown in Figures 6 through 11 to observe the relations between individual goals' preferences over the design space, and the end resulting preference surface.

As can been seen, the aggressive strategy will produce higher overall preference than a conservative strategy, and the two strategies will result in different solution design parameter values for the design: different l^*,r^* have the highest μ on the overall preference surfaces of Figures 13 and 15 (hemispherical head design), and likewise for Figures 12 and 14 (flat head

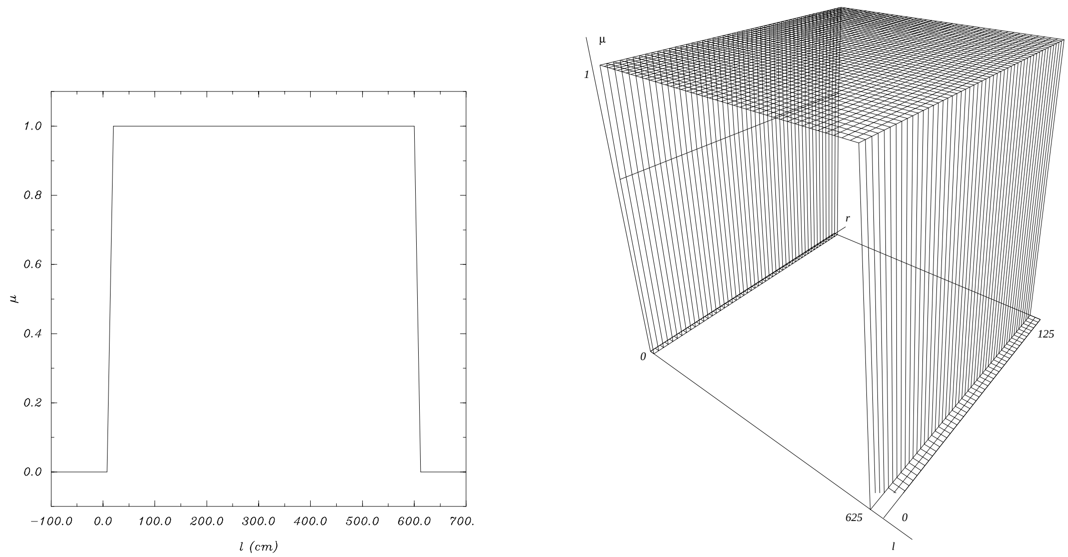

Figure 6: Length l preference.

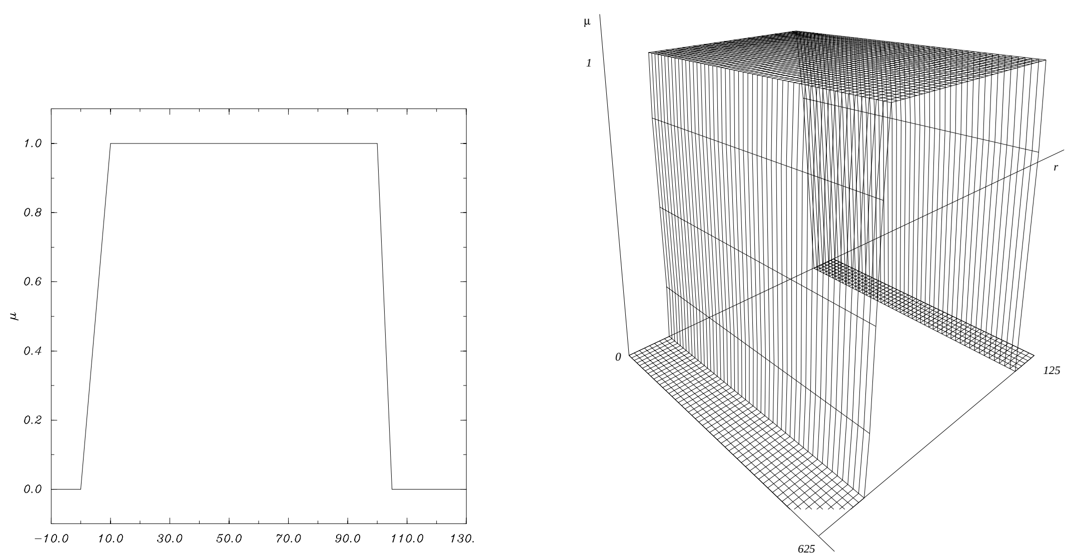

Figure 7: Radius r preference.

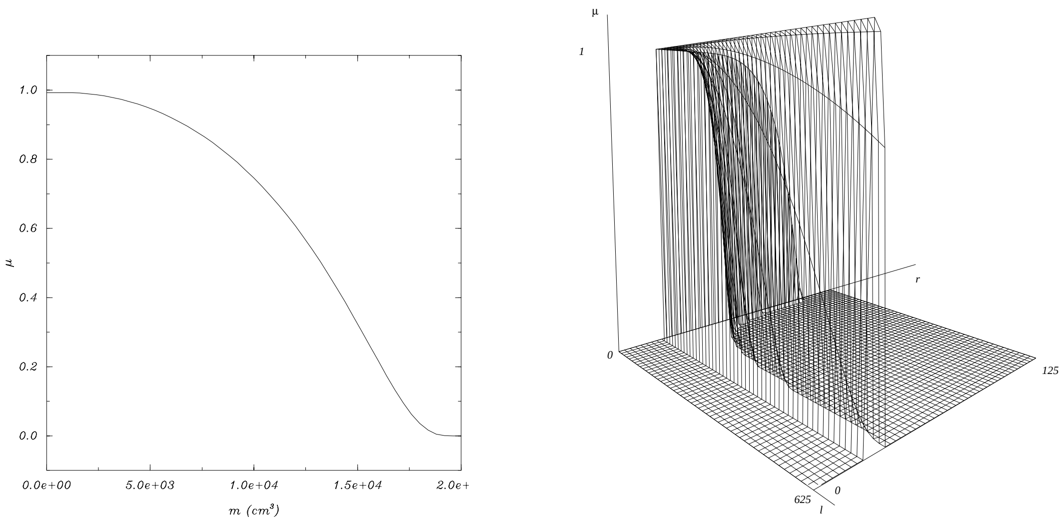

Figure 8: Metal volume m preference.

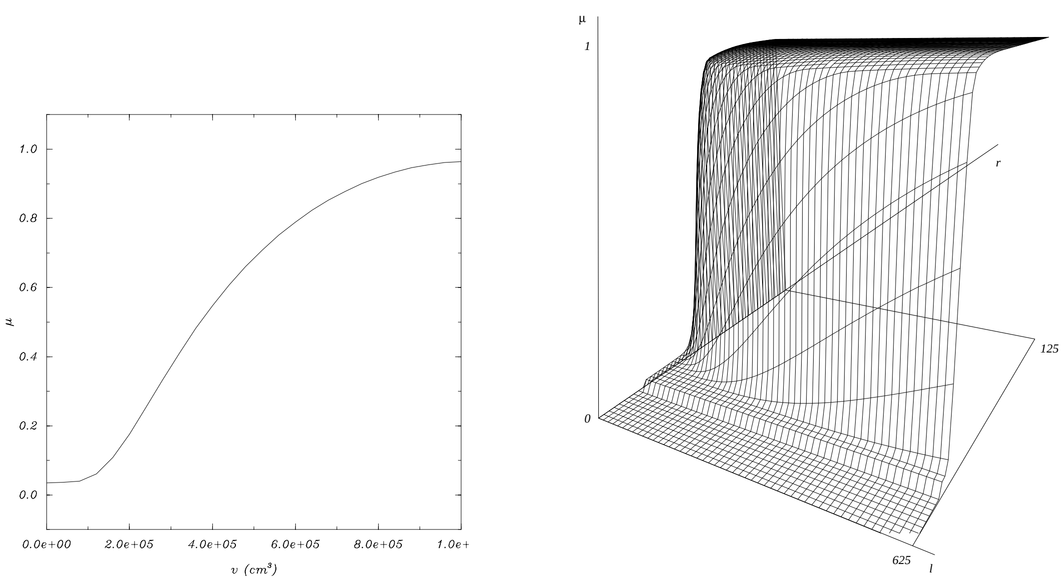

Figure 9: Capacity v preference.

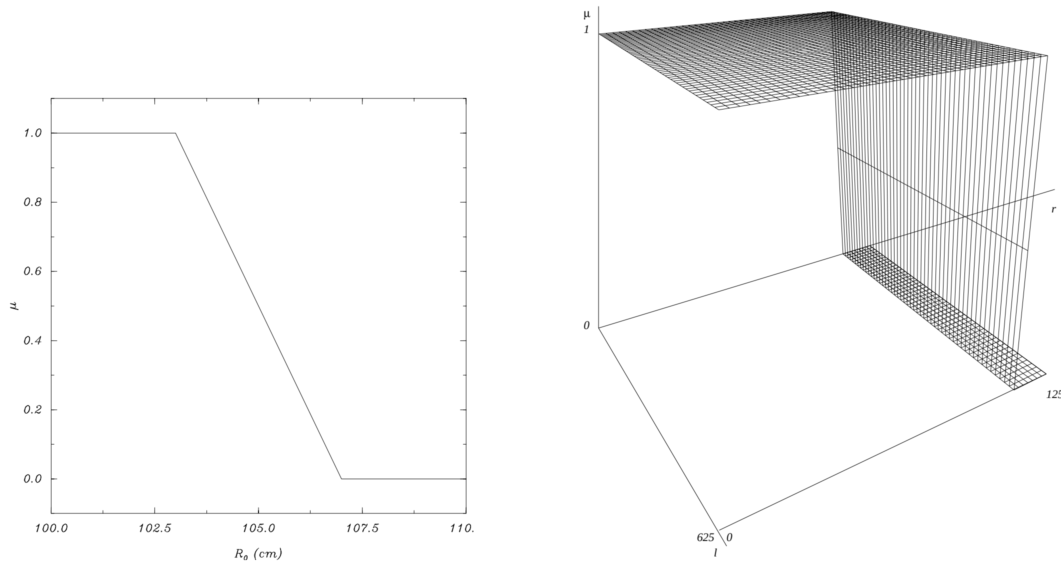

Figure 10: Outer radius R_0 preference.

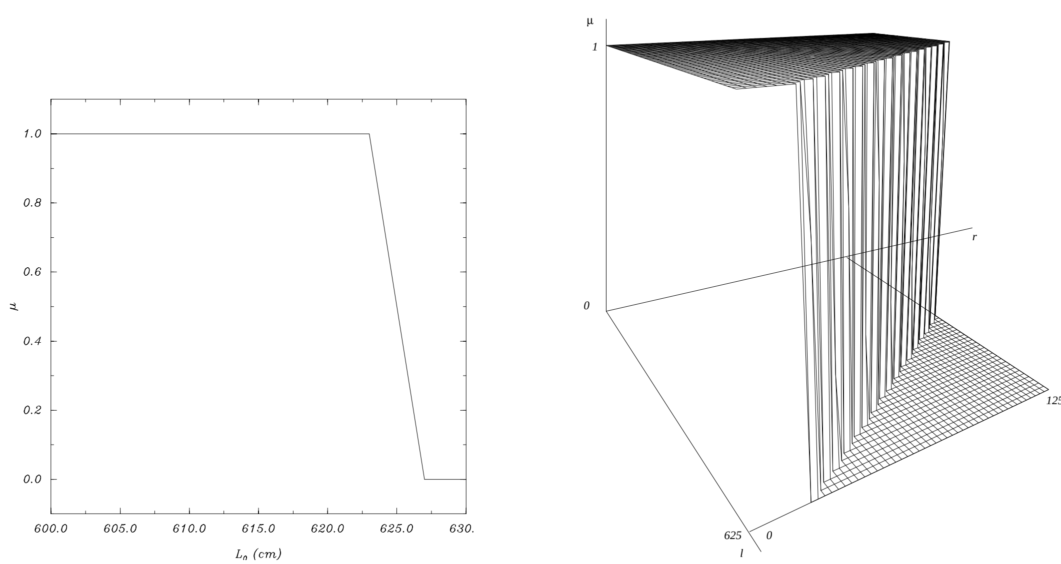

Figure 11: Outer length L_0 preference.

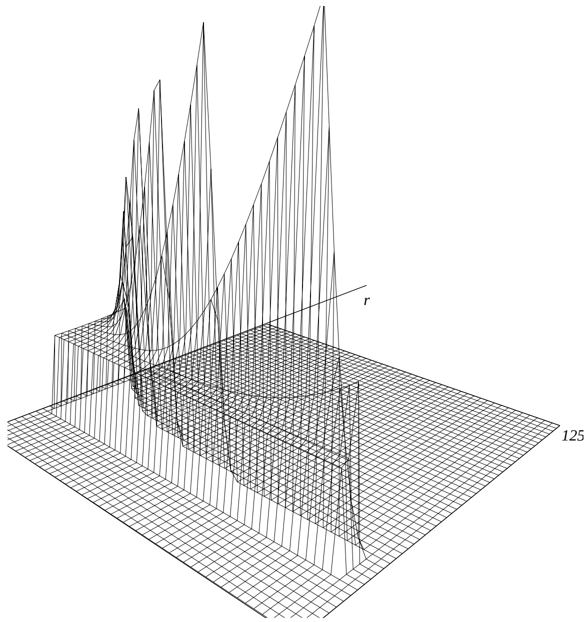

Figure 12: Flat head tank design: conservative design strategy results.

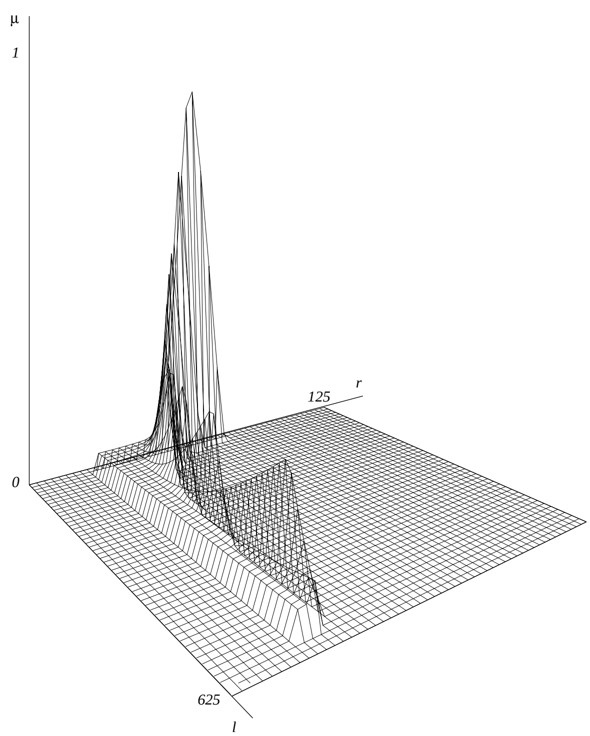

Figure 13: Hemi head tank design: conservative design strategy results.

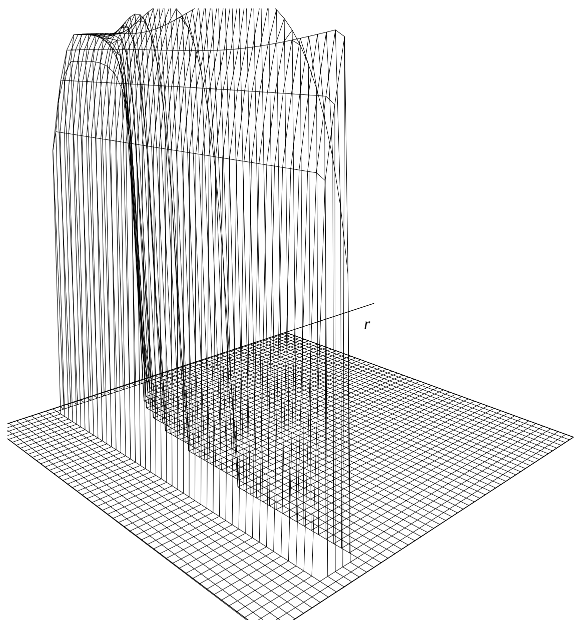

Figure 14: Flat head tank design: aggressive design strategy results.

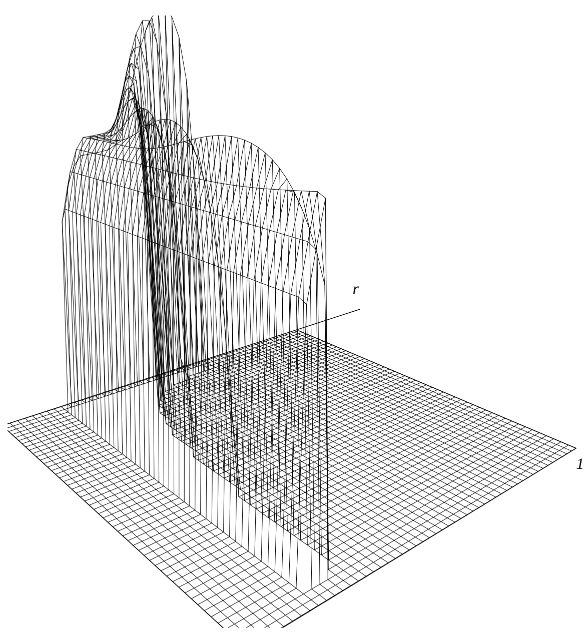

Figure 15: Hemi head tank design: aggressive design strategy results.

design). The conservative design strategy sacrificed the cost (m) to ensure the capacity (v). Designing aggressively did the reverse: reduced the cost (m) at expense of the capacity (v). Observe that, from an optimization viewpoint, both solutions are in the Pareto-optimal solution set, since both require a reduction in preference of a goal to increase another.

This differs from the results of the various problem formulations presented in Papalambros and Wilde [16]. For example, the non-linear programming formulation solves the problem by minimizing the metal volume with the rest of the goals as crisp constraints. Our formulation allows the constraints to be elastic, as shown in Figures 6 through 11, so the final design parameter values determined are different than if crisp constraints had been used. If the example had selected step functions for preference curves on the constraint performance parameters, the imprecision results would reduce to the non-linear programming solution for any strategy. This is because, with step functions for the constraint parameters, only one parameter (m) dictates the preference, and so the issue of trade-off between goals is not applicable: there is only one goal. The point of this example is to visually demonstrate the differing overall preference (shown here as surfaces) over the design space, and to demonstrate that different design trade-off strategies can entirely change the solution.

4 Conclusion

This paper presents a method for trading off multiple, incommensurate goals. The definition of "best" in light of the incommensurate goals is determined by specifying a formal, explicit design trade-off strategy.

This work permits direct comparisons of different design configuration alternatives in a formal sense: by comparing their respective overall preference ratings. This is true even if the different alternatives have vastly different physical forms, or even if the different configurations have different parameters. Further, the formalization of these strategies, introduced here, shows that strategies can guide design decisions. Two simple examples were presented to demonstrate the methodology in familiar domains. It was shown that current matrix method formalizations for preliminary design (such as Pugh's method [17], and QFD [1, 13]) use a compensating design strategy to trade-off the different features of alternative configurations. This is because they select a configuration based on the net sum of designer rankings, rather than on worst case. This research will allow designers to apply the same techniques, but with different design strategies, as appropriate.

This new methodology permits the designer to formally implement a design trade-off strategy, and incorporate subjective knowledge and experience (via preference and imprecision). This makes the designer's prejudices and strategy explicit, which can be used to help make, observe, justify, and record design decisions.

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