The Method of Imprecision Compared to Utility Theory for Design Selection Problems*

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Abstract

Two methods have been proposed for manipulating uncertainty reflecting designer choice: utility theory and the method of imprecision. Both methods represent this uncertainty across decision making attributes with zero to one ranks, higher preference modeled with a higher rank. The two methods can differ, however, in the combination metrics used to combine the ranks of the incommensurate design attributes. Utility theory resolves the multi-attributes using various well proven additive metrics. In contrast, the method of imprecision resolves by also considering non-additive metrics, such as ranking by the worst case performance or by multiplicative metrics. The axioms of utility theory are appropriate for designs where it is deemed the attributes can always be traded off, even to the point of achieving zero preference in some attributes. In the case of a design with attributes which cannot have zero preference, such as stress limits or maximum allowed cost, the method of imprecision is more appropriate: it trades off attribute levels without permitting any of them to be traded off to zero performance.

1 Introduction

In the design of engineered devices, a necessary design task is to make a selection among candidate designs or parametric values, after the design has been formalized. This can be in the preliminary concept phase where a designer makes choices over

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alternatives to pursue, or perhaps in the detail design phase where a designer makes choices over particular values to use in a design model. In the design literature, two similar methods have been proposed for this particular design task. Utility theory [22], originally developed by von Neumann and Morgenstern [22] for economic decision making, has been applied to design [9, 17, 18, 19, 20]. The Method of Imprecision [14, 24], developed by Otto, Antonsson, and Wood, is a similar method developed for engineering design. This paper will analytically compare the two methods by examining their basic axioms of usage and discuss their axioms' applicability to engineering design, restricting to cases of no confounding influences (noise). Therefore influences such as manufacturing errors or effects of agents other than the designer will not be considered here, and are discussed elsewhere [16].

Utility theory [22] is an analytical method for making a decision concerning an action to take, given a set of multiple criteria upon which the decision is to be based. Originally developed for economic decision making, it has been extensively developed with a rich literature [9, 17]. With utility theory, a basic set of axioms are proposed to restrict the way by which a designer should make preferential judgments among design options, represented as a set. If these restrictions are believed, then it can be shown that the formalism adopted by utility theory is the only method consistent with these restrictions. Thus, if the restrictions are believed, then a designer must use utility theory when making decisions.

In contrast, the method of imprecision (presented in detail previously [12, 13, 14, 15, 23, 24, 25]) is a method developed to formally negotiate a design problem as desired by a designer. Originally developed as a method for providing preliminary metrics of performance [23, 24, 25], it has been expanded to consider selection problems [14], and also to consider iterative design [12, 13, 15]. Only the selection problem is considered here, since of these, selection is the area utility theory addresses. With the method of imprecision, a core set of axiomatic restrictions are proposed governing a designer's preferential judgments over design options, where again, as with utility theory, the design options are represented as a set. Then additional restrictions are provided which imply particular combination metrics. Thus, as with utility theory, if these restrictions (which are different from utility theory's) are believed, then a designer must use the method of imprecision when making decisions.

For selection problems, a degree of formalization of the design problem is required, regardless of which method is used. A set of design configuration options must be known, and will be called the *design parameter space*, DPS. A point d_i in the DPS is a representation of a design configuration (perhaps as a vector). The decision over which design configuration to use is thus a selection for the optimal d^* in the DPS. Optimal must be defined. It is assumed to be determined from well identified availability and performance criteria. The set of possible performance criteria (or attribute) values will be called the *performance parameter space*, PPS. A point p_j in the PPS is an evaluated performance (a vector for multi-criteria problems), evaluated at a point d in the DPS. Henceforth, points within the sets will be denoted with lower case letters with subscripts, and components of the points with superscripts.

The next section will discuss the axioms of the method of imprecision. Then the following section will similarly discuss the axioms of utility theory. Finally, the two will be compared using design methodology criteria.

2 The Method of Imprecision

Given the design and performance spaces, the intent is to select a d in the DPS, based on availability and performance. As presented in [14, 24], availability concerns can be directly stated on the design configuration components d^i. Desired performance concerns can be directly stated on the performance criteria components p^j. Some of these preferential specifications can be made without considering any of the other parameters. Material stress is a performance parameter which a designer simply knows must be less than the yield stress. Cost might have to be contained below a maximum. Other parameters might have to be considered simultaneously for a designer to make preferential statements. If a hard-to-get part is much cheaper than a readily available part, the availability and cost are considered together when making preferential statements. Thus, in the method of imprecision, the parameters (d^i and p^j) must be grouped into N *goals* x^k over which preferential statements can be made, independent of the remaining goals. The complete set of values which x^k can assume will be denoted by the set X_k. If no such divisions of the DPS and PPS into separate spaces exist, then the (DPS $\times$ PPS) is a single goal X.

In the method of imprecision, the preferential statements for values (to attain a goal) are formalized. This is done by constructing a scale converting the preferential statements into a [0,1] numerical rank. This rank is called a *preference* μ. It is a map from a goal X_k to [0,1], preserving the designer's preferential order over X_k.

Such a map can be constructed, if the designer can elicit the preferential ordering over X_k, (an assumption which must be met to be able to apply the method of imprecision). A rigorous method for doing so is the lottery method [11], by asking the designer to first identify over all X_k the least and most preferred points ${x^k}_0$ and ${x^k}_1$. These are given numerical ranks 0 and 1. Then for every other point ${x^k}_i$ in X_k, one asks a lottery question:

> "On a scale from 0 to 1, what is your numerical belief μ you are indifferent between receiving 1) ${x^k}_i$ or 2) ${x^k}_1$ with certainty μ and ${x^k}_0$ with certainty $1-\mu$?"

This construction provides a numerical rank over X_k which preserves the preferential order.

This method provides a rigorous basis for constructing performance ranks of a design configuration based on availability and performance. In a typical design process, however, it is usually not used explicitly. In many cases, a designer can simply directly state preferences for goals, since many reflect customer requirements which have been expressed to the designer. While such direct methods have been shown to produce inconsistent results [2, 3], it is argued that an iterative design process is iterative to eliminate such inconsistencies. A designer is free to change preference ranks in light of the implied results of the initial specification, should

Table 1: Method of Imprecision General Axioms

$\mathcal{P}(0,\ldots,0)=0 \quad \mathcal{P}(1,\ldots,1)=1$	(boundary conditions)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) \le \mathcal{P}(\mu^1,\ldots,\mu^{k\prime},\ldots,\mu^N)$ iff $\mu^k \le \mu^{k\prime}$	(monotonicity)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) = \lim_{\mu^k \to \mu^{k\prime}} \mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N)$	(continuity)
$\mathcal{P}(\mu^1,\ldots,0,\ldots,\mu^N)=0$	(annihilation)
$\mathcal{P}(\mu,\ldots,\mu)=\mu$	(idempotentcy)

the results be unacceptable. In any case, the basic lottery method does provide one rigorous primary method for constructing numerical preferences, should such simpler methods not be useful.

The overall result desired in selection problems, however, is to provide a single numerical rank of each design configuration. That is, the N ranks must be combined in some fashion to provide a single overall numerical rank. In the method of imprecision, this is directly formalized into the statement

$$\mu(d) = \mathcal{P}(\mu(x^1),\ldots,\mu(x^N)), \tag{1}$$

where $\mathcal{P}$ is some combination function, p is evaluated at d, and $(x^1,\ldots,x^N)$ consists of the groupings of the components of d and p, where d and p might be represented with components $(d^1,\ldots,d^n)$ and $(p^1,\ldots,p^m)$ in particular bases for multi-component and multi-attribute problems, for example. The design problem is thus formally transferred into finding the optimal d^* in the DPS, where

$$\mu(d^*) = \sup\{\mu(d) \mid d \in \text{DPS}\}. \tag{2}$$

The formal question now becomes what function $\mathcal{P}$ to use. This directly reflects how the various design goals should be traded off. It has been proposed in [14] that any method of trading off design goals should obey the restrictions in Table 1. The reader is referred to [14] for a discussion of these restrictions. Of importance for this comparison, however, is the third restriction, annihilation. This states that if any goal of a design fails, then the whole design fails, regardless of the performance achieved in any other goal. Thus, if material stress is excessive, no amount of decrease in cost will compensate for this fact. This restriction is essential for any multi-attribute engineering decision. If it is not satisfied, then a decision could be made in which a goal is not satisfied (such as, for example, material stress) but is mathematically compensated for by large preference of another goal (such as, for example, cost). This is not reasonable for engineering design, as others have also argued [1, 21].

The restrictions in Table 1 are deemed necessary for all design, but they are not sufficient to restrict $\mathcal{P}$ to any operator. More restrictions are needed. Consider rating a design by its most poorly performing aspect. Such a *design strategy* [14] for resolving the multiple goals can be directly formalized over [0,1] ranks as the *min* function. That is,

$$\mu(d) = \min\left\{\mu^1,\ldots,\mu^N\right\} \tag{3}$$

Table 2: *Min* Combination Metric Axioms

$\mathcal{P}(0,\ldots,0)=0 \quad \mathcal{P}(1,\ldots,1)=1$	(boundary conditions)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) \le \mathcal{P}(\mu^1,\ldots,\mu^{k\prime},\ldots,\mu^N)$ iff $\mu^k \le \mu^{k\prime}$	(monotonicity)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) = \lim_{\mu^k\to\mu^{k\prime}} \mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N)$	(continuity)
$\mathcal{P}(\mu^1,\ldots,0,\ldots,\mu^N)=0$	(annihilation)
$\mathcal{P}(\mu,\ldots,\mu)=\mu$	(idempotentcy)
$\mathcal{P}(\mu^1,\ldots,a,\ldots,b,\ldots,\mu^N)=\mathcal{P}(\mu^1,\ldots,b,\ldots,a,\ldots,\mu^N)$	(commutativity)
$\mathcal{P}(\mathcal{P}(\mu^1,\ldots,\mu^{N-1}),\mu^k)=\mathcal{P}(\mu^k,\mathcal{P}(\mu^1,\ldots,\mu^{N-1}))$	(associativity)
$\mathcal{P}(1,\ldots,1,\mu^k,1,\ldots,1)=\mu^k$	(identity)

Table 3: Product Combination Metric Axioms

$\mathcal{P}(0,\ldots,0)=0 \quad \mathcal{P}(1,\ldots,1)=1$	(boundary conditions)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) \le \mathcal{P}(\mu^1,\ldots,\mu^{k\prime},\ldots,\mu^N)$ iff $\mu^k \le \mu^{k\prime}$	(monotonicity)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) = \lim_{\mu^k\to\mu^{k\prime}} \mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N)$	(continuity)
$\mathcal{P}(\mu^1,\ldots,0,\ldots,\mu^N)=0$	(annihilation)
$\mathcal{P}(\mu,\ldots,\mu)=\mu$	(idempotentcy)
$\mathcal{P}(\mu^1,\ldots,a,\ldots,b,\ldots,\mu^N)=\mathcal{P}(\mu^1,\ldots,b,\ldots,a,\ldots,\mu^N)$	(commutativity)
$\mathcal{P}(\mathcal{P}(a,b),\mathcal{P}(c,d))=\mathcal{P}(\mathcal{P}(a,c),\mathcal{P}(b,d))$	(bi-symmetry)
$\mathcal{P}(\mu^1,\ldots,\mu^k,\ldots,\mu^N) < \mathcal{P}(\mu^1,\ldots,\mu^{k\prime},\ldots,\mu^N)$ iff $\mu^k < \mu^{k\prime}$	(strictness)

Table 2 lists a set of necessary and sufficient restrictions required to restrict $\mathcal{P}$ to *min*, as proven in [4]. If a designer believes these restrictions, then the designer must use *min* when making decisions.

An important additional restriction used to restrict $\mathcal{P}$ to *min* is the identity restriction. If a design is perfect except for a single goal, then the identity restriction states the design should be rated by that goal.

Instead of rating a design by its most poorly performing aspect, a design might instead be rated by a compensation among the goals. Thus, a highly performing goal can compensate for a different goal exhibiting less performance. Such a compensating metric must be constructed in a manner consistent with Table 1 to ensure, for example, that the annihilation condition is satisfied. One method is to use a normalized product,

$$\mu(d) = \left(\mu^1,\ldots,\mu^N\right)^{\frac{1}{N}} \tag{4}$$

This metric trades off the goals to cooperatively improve the design. Again, if this metric is to be used, the restrictions it satisfies should be known. Table 3 lists the restrictions the normalized product satisfies.

The final additional important restriction is the strictness condition: if any goal changes at all, then the overall metric must change. Thus, any goal can compensate for any another. Also, the distributivity condition of the *min* is relaxed into a bi-symmetry condition. This reflects that equal number of goals should carry the same capacity to trade off, rather than being independent of the number of goals.

These are two basic metrics (trade-off strategies) used within the method of imprecision. Of course, any metric consistent with Table 1 is consistent with the theory. For example, a metric could be constructed using the *min* and *product* over different subsets of the goals, which would still be consistent with Table 1. Also, weighting factors are commonly used in engineering design, due their ease of interpretation, sufficiency in functionality, and ease of construction from methods such as customer questionnaires. In the method of imprecision, weighting functions can also be easily incorporated. The reader is referred to [14].

3 Utility Theory

Utility theory is very similar to the method imprecision, as applied to decision problems. It also requires the same level of formalization: a DPS representing the design configurations and a PPS representing the performances. In the case of utility theory, however, no preferences are assigned directly to the design parameters d^i. The determination is made based solely on performance, and the availability of the design configurations is not considered.

The determination of a preference μ is made again using a lottery method, as was done in the method of imprecision. Indeed, the lottery method was developed as a part of utility theory. Also, again the objective is to find the design configuration maximizing the overall preference (utility), Equation 2. With utility theory, however, the performance parameters are not split into independent goals, as was done in the method of imprecision. Utility theory advocates maintain that this is impossible, because there is only one goal X which is the PPS, having components the same as the PPS [17].

Thus, with utility theory, one theoretically applies the lottery method over then entire PPS, even if the PPS has additional vector space structure which is then not used. When considering a PPS with components, however, the theory adds sufficient restrictions such that one can (even though the components of the PPS $= X$ are not independent) make preference specifications on each component separately [5, 7, 10, 11, 17].

That is, the different performance parameters are assumed to be such that one can make specifications of one component (say p^1) with the remaining components (say $p^2, \ldots, p^m$) fixed at particular values. The theory adds sufficient restrictions such that one can extrapolate away from the fixed component values $(p^2, \ldots, p^m)$, and still have the preferences determined over the performance parameter (p^1) remain valid.

The restrictions further ensure that one can then combine the component performance preference ranks with a linear combination, thus arriving at a single numerical rank for the performance parameter vector. Through composition of the performance parameter map one maps the preference back onto the design space to provide a rating for each design configuration: $\mu(d) = \mu(p)$ where $p = f(d)$, $d \in$ DPS, $p \in$ PPS.

Under utility theory, two methods have been developed for separating the PPS into m preference specifications which must then be combined (where there are m

components to the PPS). The first method provides necessary and sufficient restrictions such that the combination procedure is simple addition: $\mathcal{P}$ is additive. These restrictions are shown in Table 4. They are shown in two sections; the first section lists the restrictions required of the designer when stating preferences: restrictions required of $\mu(p)$. If this first set is satisfied, then $\mu(p)$ can be decomposed into components, and

$$\mu(p) = \sum_{j=1}^{m} \mu(p^j) \frac{1}{m} \tag{5}$$

Actually, the restrictions only constrain μ to being linear in the entries. Choosing a base point and scale of [0,1] provides a complete set of restrictions. Also shown in the second set for reference are the implied restrictions that $\mathcal{P}$ (normalized addition) must then obey.

The reader is referred to [11, 17] for justification of these restrictions. Notice that four restrictions assumed by the method of imprecision, shown in Table 1 (boundary conditions, monotonicity, continuity, and idempotentcy) are satisfied by the additive form of utility theory.

The additive form does not, however, satisfy the annihilation condition used in the method of imprecision and shown in Table 1. This is caused by the assumed Archimedian property of utility theory, shown in Table 4. This restriction demands that any decrease in overall preference caused by changes in the performance of one parameter must *always* be able to be compensated for by an increase in performance in any of the other parameters. Thus, it is assumed any decrease in preference caused by, for example, an increase in material stress levels, can always be compensated for by an appropriate amount of decrease in cost. The difficulty is in the "always:" it is actually only true up to the yield point of the material in question, in this example. The Archimedian property of utility theory directly implies that annihilation will not be satisfied. Annihilation is adopted by the method of imprecision for all metrics. This is a prime difference between the methods.

The developers of utility theory acknowledge that the additive form is too restrictive even for simple decision making problems [17]. A less restrictive version is also available, known as the multi-linear form. Here, Thompsen's condition is dropped, leaving a multi-linear representation, but whose attribute coefficients remain to be determined. These are determined by asking relative trade-off questions between the performance parameters, which are then extrapolated across the whole PPS. The overall preference for a performance parameter takes the form

$$\mu(p) = \frac{1}{k} \left(\prod_{j=1}^{m} (k k^j \mu(p^j) + 1) - 1 \right) . \tag{6}$$

where k and k^j are the coefficients to be determined. This form also requires the Archimedian restriction. Thus again, the annihilation condition adopted by the method of imprecision is not satisfied. The commonalities, however, are that the first three assumptions of Table 1 in the method of imprecision (boundary conditions, monotonicity, continuity) are satisfied by the multi-linear form.

Table 4: Additive Combination Metric Axioms

$\mu(p^1, \ldots, a, \ldots, p^m) \geq \mu(p^{1\prime}, \ldots, a, \ldots, p^{m\prime})$ then $\mu(p^1, \ldots, p^m) \geq \mu(p^{1\prime}, \ldots, p^{m\prime})$	(mutual preference independence)
if $\mu(\ldots, p^i_1, \ldots, p^j_2, \ldots) = \mu(\ldots, p^i_2, \ldots, p^j_3, \ldots)$ and $\mu(\ldots, p^i_3, \ldots, p^j_1, \ldots) = \mu(\ldots, p^i_2, \ldots, p^j_3, \ldots)$ then $\mu(\ldots, p^i_3, \ldots, p^j_2, \ldots) = \mu(\ldots, p^i_2, \ldots, p^j_3, \ldots)$	(Thompsen's condition)
$\exists\ p^i$: if $\mu(\ldots, \beta, \ldots, a, \ldots) \geq \mu(\ldots, p^i, \ldots, b, \ldots) \geq \mu(\ldots, \alpha, \ldots, a, \ldots)$ then $\mu(\ldots, p^i, \ldots, a, \ldots) = \mu(\ldots, p^i, \ldots, b, \ldots)$	(restricted solvability)
$\exists\ n$: $\mu(\ldots, na^i, \ldots, na^j, \ldots) \geq \mu(\ldots, b^i, \ldots, b^j, \ldots)$	(Archimedian Property)
$\exists a, b$: $\mu(p^1, \ldots, a, \ldots, p^m) \geq \mu(p^1, \ldots, b, \ldots, p^m)$	(essentiality)
$\mathcal{P}(0, \ldots, 0) = 0 \quad \mathcal{P}(1, \ldots, 1) = 1$	(boundary conditions)
$\mathcal{P}(\mu^1, \ldots, \mu^k, \ldots, \mu^N) \leq \mathcal{P}(\mu^1, \ldots, \mu^{k\prime}, \ldots, \mu^N)$ iff $\mu^k \leq \mu^{k\prime}$	(monotonicity)
$\mathcal{P}(\mu^1, \ldots, \mu^k, \ldots, \mu^N) = \lim_{\mu^k \to \mu^{k\prime}} \mathcal{P}(\mu^1, \ldots, \mu^k, \ldots, \mu^N)$	(continuity)
$\mathcal{P}(\mu, \ldots, \mu) = \mu$	(idempotentcy)
$\mathcal{P}(\mu^1, \ldots, a, \ldots, b, \ldots, \mu^N) = \mathcal{P}(\mu^1, \ldots, b, \ldots, a, \ldots, \mu^N)$	(commutativity)
$\mathcal{P}(\mathcal{P}(a, b), \mathcal{P}(c, d)) = \mathcal{P}(\mathcal{P}(a, c), \mathcal{P}(b, d))$	(bi-symmetry)
$\mathcal{P}(\mu^1, \ldots, \mu^k, \ldots, \mu^N) < \mathcal{P}(\mu^1, \ldots, \mu^{k\prime}, \ldots, \mu^N)$ iff $\mu^k < \mu^{k\prime}$	(strictness)

4 Comparing the Methods

Given these two theories about combining preferences, the axioms forming the theories' basis can be compared for design purposes. Five characteristics important for design are considered here.

The first consideration is the strength of the axiomatization. Utility theory is better in this respect, in that the axioms used to restrict the combination to the additive metrics are placed on the PPS itself, not on the constructed preferences. The restrictions are on μ over the PPS, not on the metric $\mathcal{P}$ which combines the various component $\mu(p^j)$s. Theoretically, this is preferable, since one would like axioms that pertain to the selection problem, not in how the problem is modeled. On the other hand, the method of imprecision is still new, and not fully developed mathematically. We believe that there are exact equivalent statements of each restriction in Table 1 over the (DPS $\times$ PPS), but as of now this remains unexplored.

A second consideration is the usefulness of the resulting theory for design purposes. Utility theory was developed for the management of decisions, not for engineering design. Economists generally believe every aspect of a decision can always be directly translated into associated cost, measured in dollars. Dollars are additive. Aspects which are not additive, or which cannot be "bought off," are not deemed possible. Clearly this is not the case in engineering design, as others have argued [1, 21]. For example, given a fixed material, the tensile strength limits cannot be exceeded no matter the reduction in the design's cost. Material stress simply cannot always be traded off in a compensating fashion.

The axiom of utility theory which creates this demand, that a gain in any aspect must be able to compensate for any loss in any other aspect, is the Archimedian property. This prevents the annihilation condition from being satisfied: the two axioms cannot be simultaneously satisfied. This implies utility theory will not permit a worst case analysis, which is required in many instances in engineering design.

However, the two theories have much in common. Both use [0,1] real valued metrics, and are monotonic continuous in the goals. Thus both theories satisfy the first three axioms in Table 1. But far more importantly, both can rely on the same lottery method to theoretically formulate the basic numerical preference over a set X (when there is no noise in the problem, as delineated in the introduction). Thus, the two methods are identical when there is only one goal: the method of imprecision reduces to utility theory.

Another consideration is to compare the theories philosophies. Utility theory operates by having a designer accept the axioms, and solve the design problem by formulating preferences over the criteria, and then optimizing the "utility," as combined with numerical methods consistent with the initial axioms accepted. Any other way of determining a solution is said to be "irrational" [6, 7].

The philosophy behind the method of imprecision, on the other hand, is to let the designer select from a variety of overall combinations, to observe the results, without dictating which metric is "correct." The method of imprecision only attempts to make the overall combination explicit, so the designer will realize which goal resolving strategy is being used. An argument against this is that perhaps the method of imprecision is then "pandering" to the designer's ignorance: allowing the designer to make statements and combine them before having determined why they are appropriate, and if the results as satisfactory, allowing the designer to stop with a "sub-optimal" solution.

The defense against this criticism is that one must give the designer credit for some intelligence. The designer must determine what is "optimal." Any method should allow designers to iteratively determine this by choosing which trade-off strategy appears most appropriate, and to allow the designer to modify any and all of these initial choices. The method of imprecision is intended for *design* problems. Design problems *are* commonly solved in an iterative manner, not usually with a single formalization and subsequent optimization. In an iterative design process, a designer makes determinations without complete understanding, thus enabling the designer to (ultimately) form a more complete understanding. For example, if the *product* resolution returns a poor solution on one performance component, the designer can the switch strategies to a *min* to improve that aspect. This is the nature of iterative design. Thus there is an inherent philosophical difference between the method of imprecision and utility theory: utility theory operates by assuming an *a priori* axiomatization, constructing preferences, and then producing an "optimal" solution consistent with the axiomatization. The method of imprecision, on the other hand, was developed specifically to include the iterative nature of design decision making, and initial preferences are intended to be modifiable. However, in practice, utility theory is also used iteratively [7, 8].

A final consideration to contrast the two methods is by their overall relevance to the complete design task. Our discussion thus far has focused on the designer's task of choosing particular design parameter values, which is important, yet typically brief. A more central concern is determining the DPS and PPS initially. For example, what functions must the design satisfy? What tools can the designer use to formulate this problem? The method of imprecision does provide some assistance in formalizing the problem, by providing indications on which aspect of the design is most important to the different performance metrics (see, for example, the discussion of the γ-level measure in [24]). This provides a designer with indications of what part of the DPS requires further design consideration.

Even when a DPS and PPS have been formalized, however, the design task, as it occurs in practice, is generally remains iterative: making partial selections, and observing the effects on performance. Formalization of design iteration is the problem addressed by the complete method of imprecision (not restricted to selection problems). The reader is referred to [12, 13, 14, 24], which discuss how preferences over some parameters can be propagated to the unspecified parameters, to observe the effects. This provides a formalization of the iterative nature of engineering design activity.

5 Conclusions

The method of imprecision and utility theory represent designer uncertainty in choice with zero to one ranks. The combination metrics of these ranks across the various goals of a design problem are different. Utility theory has a more complete axiomatization, covering restrictions on preferential statements over the PPS. The method of imprecision is not as complete in this respect. The axioms of utility theory are appropriate for design problems in which the attributes can always be traded off. In the different case of a design with attributes which cannot be traded off, such as a design involving material stress limits or maximum allowed cost, the method of imprecision is more appropriate. Finally, the problem addressed in this paper, (that of parameter value selection) is an important but minimally time consuming task in a design process. A generally more time consuming task is the development of the design problem formalization. This is by definition not computational, it is a formalization task that must be completed prior to computation. Computational assistance can be provided, however, using methods of partial specification and computation, as developed in the method of imprecision and discussed in [13, 14].

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