Modeling Imprecision in Product Design *

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Abstract

A method for representing and manipulating imprecise and vague information in engineering design is described. Designers and customers preferences are captured with Fuzzy sets. Formal methods for including noise, trade-off strategies and design iteration are included.

Introduction

Imprecision and vagueness are intrinsic aspects of engineering design. If (at the start of a design process) a proposed solution were neither imprecise nor vague, its description would be precise and it would therefore be a completed design. While (stochastic) uncertainty typically remains in a completed design description (e.g., dimensional tolerances), the nominal desired dimensions are precise. However, much of the early description of a design concept (physical dimensions, material properties, etc.) is vague and imprecise. Engineering design is essentially the process of reducing the *imprecision* in the description of solution concepts.

Imprecision occurs throughout the product design process. It is most easily, and naturally, observed in the early stages when a designer (or group of designers) is articulating a potential solution concept. The "back of the envelope" sketch will not include precise dimensions or other attributes, but is almost universally described imprecisely (e.g., "This diameter will be *about* 2 cm."). This imprecision can be represented by designers preference and customers preference.

Our approach is to ask the designers and customers their preferences for various aspects of the design. They indicate their preference (ranked from 0 to 1) for each value of each parameter describing the design. We describe the customers' needs using Performance Parameters, which define the desired performance of the design. We denote each performance parameter as p_j, and the associated preference function is denoted $\mu(p_j)$. This collection of Performance Parameters spans the Performance Parameter Space (PPS). We describe the designers' preferences as Design Parameters, which include the designers' experience and expertise, as well as availability, manufacturability, and other attributes of the design. Similarly we denote each design parameter as d_i, and the associated preference function is denoted $\mu(d_i)$. This collection of Design Parameters spans the Design Parameter Space (DPS).

The preference information is usually (but not always) a convex Fuzzy Number. Thus, we use the mathematics of Fuzzy Calculus to operate on these imprecise descriptions of the design, but we do not treat this information as Fuzzy membership, nor do we perform fuzzifying or de-fuzzifying operations, or logical operators on fuzzy sets. Rather, we use this imprecise preference data to perform the usual engineering computations encountered in design to map the preferences of the designers onto the Performance Parameter Space, and also to map the preferences of the customers onto the Design Parameter Space. This provides the designer with the ability to trade-off the many (and typically incommensurate) aspects of a design in an understandable way.

As mentioned above, stochastic uncertainty (noise) also is typically present in engineering design, and can be characterized by Noise Parameters. A noise parameter n_k might be the possible positioning of an operator switch, and so the alternatives may be finite. Alternatively, n_k might be a value of a manufacturing error on a design parameter, and so the Noise Parameter Space (NPS) may have a continuum of possibilities. The approach we describe here can also include these effects, and can show how the level of noise present in a design affects the customers and designers preferences, and the trade-offs made in design decision-making.

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Table 1: Overall Preference Resolution Axioms.

$\mathcal{P}(0,\ldots,0)=0 \qquad \mathcal{P}(1,\ldots,1)=1$	(boundary conditions)
$\forall\ k, \mathcal{P}(\mu_1,\ldots,\mu_k,\ldots,\mu_N) \leq \mathcal{P}(\mu_1,\ldots,\mu'_k,\ldots,\mu_N)$ iff $\mu_k \leq \mu'_k$	(monotonicity)
$\forall\ k, \mathcal{P}(\mu_1,\ldots,\mu_k,\ldots,\mu_N) = \lim_{\mu'_k \to \mu_k} \mathcal{P}(\mu_1,\ldots,\mu'_k,\ldots,\mu_N)$	(continuity)
$\mathcal{P}(\mu_1,\ldots,0,\ldots,\mu_N)=0$	(annihilation)
$\mathcal{P}(\mu,...,\mu)=\mu$	(idempotentcy)

Engineering Design with Imprecision

Representing and manipulating imprecision can be used to *assist* the engineering design process, under the hypothesis that doing so will allow the designer to make faster and more-well-informed decisions. At the start of the process, a designer suggests possible solutions. These are subsequently transformed into formal candidate solution models upon which computations can be performed. Having determined conceptual models of the proposed candidates, the designer then specifies the design parameters imprecisely. Having performed this formalization, the extension principle is invoked to calculate the imprecise achievable performance. This calculation is performed over the *entire set* of imprecise values specified, allowing the designer to make better evaluations. After having made the performance parameter preference calculations, the designer can observe the imprecise performance achievable and proceed to judge the candidates.

Combining Imprecision in Design

One of the most challenging aspects of design decision-making is to compare different design alternatives. This is especially difficult because each alternative typically has many different attributes (such as weight, size, cost, etc.) that are not immediately comparable. Aggregating these attributes to arrive at a rating for each design (and trading-off attributes with an aggregation to arrive at the best rating for each alternative) is difficult. The traditional approach is to use a weighted sum of attribute values. This approach, however, has difficulties, as we will show below.

With the method of imprecision, incommensurate attributes can be aggregated by combining the unrelated preference specifications: namely by combining the μ_k. An *overall preference* is a map $\mathcal{P}$:

$$\mathcal{P} : [0,1]^N \to [0,1] \tag{1}$$

which reflects a designer's degree of overall satisfaction for a design. With this map the configuration among the options which best satisfies a designer (here denoted as d^*) is:

$$d^* : \mu(d^*) = \sup\{\mathcal{P}(\mu_1,\ldots,\mu_N) \mid d \in \text{DPS}\}. \tag{2}$$

The question is which different aggregation functions are appropriate to use as $\mathcal{P}$ for engineering design. Based on product development criteria, we have proposed restrictions that all functions (to use as connectives, or *metrics*) must be consistent with for design purposes [4]. These are shown in Table 1. This set of restrictions confines $\mathcal{P}$ to a mixed connective. This means any decision made by the designer with $\mathcal{P}$ will result in ratings which are bounded by the worst and best ratings within a design. Table 1 is a list of necessary requirements and so defines many metrics, more are discussed in [4]. Two in particular are important in design, reflecting whether a designer wishes to design subject to the worst case, or design with compensation among the goals.

Suppose a designer wishes to trade off to improve the lower performing goals when selecting a configuration d^*. Then to improve a design, there must be an increase in the preference level of the goal whose preference is lowest. This means the method to use for combining the multiple preferences is $\mathcal{P} = min$.

$$\mu(d) = \min\{\mu_1,\ldots,\mu_N\} \tag{3}$$

In this case, there are additional assumptions which must be made to restrict $\mathcal{P}$ to *min*, which a designer must consider. These additional restrictions have been developed elsewhere [2, 4]. Attributes aggregated

Table 2: Uncertainty integral restrictions.

1	$\int_{NPS} \chi_N(n)\, dg = g(N)$	(identity)
2	$\int_{NPS} \alpha\, dg = \alpha$	(linear)
3	$f_1(n) \geq f_2(n) \Rightarrow \int_N f_1(n)\, dg \geq \int_N f_2(n)\, dg$	(increasing in f)
4	$N_j \supseteq N_k \Rightarrow \int_{N_j} f(n)\, dg \geq \int_{N_k} f(n)\, dg$ or $N_j \supseteq N_k \Rightarrow \int_{N_j} f(n)\, dg \leq \int_{N_k} f(n)\, dg$.	(monotone in N)

with $\mathcal{P} = min$ are said to be combined with a non-compensating trade-off. The term "non-compensating trade-off" derives from the fact that overall preference for the design is dictated by the attribute with the lowest preference. No matter how high the preferences for the other attributes, they cannot compensate for the lowest one.

The non-compensating trade-off strategy is not always appropriate. In some circumstances higher preferences for some attributes may legitimately compensate for lower preferences in other attributes. This can be accomplished with the use of a product:

$$\mu(d) = \left(\prod_{k=1}^{N} \mu_k \right)^{\frac{1}{N}} \tag{4}$$

Equation (4) allows higher performing goals to compensate for lower performing goals (in terms of preference). Again, additional restrictions must be accepted for a designer to use this combination, as developed in [4]. Attributes aggregated with $\mathcal{P} = product$ are said to be combined with a compensating trade-off. The term "compensating trade-off" derives from the fact that overall preference for the design is determined by attributes with higher preference compensating for attributes with lower preference. This trade-off strategy can be particularly appropriate in situations where a small decrease in the preference of the lowest preference attribute will greatly increase the preference for other attributes. This is also only appropriate where decreasing the lowest preference does not violate a constraint.

Thus we have two different strategies and formalizations for considering multiple goals in a design. These can be extended to considering different importances for each goal, for example, or to combinations of them on sub-goals. In general, however, the combination must remain consistent with the restrictions in Table 1. We now turn to discussing effects of uncontrollable effects (noise) in design.

Uncontrollable Variables in Design (Noise)

Design parameters represent values that the designer is free to choose to best satisfy the performance requirements of the design, such as dimensions and material properties. Noise parameters are uncontrollable effects that a designer does not have direct choice over. Since the values cannot be chosen by the designer, and instead are dictated by random or operational effects, they must all be included in the design calculations carefully so that the effect of noise is properly included in design trade-offs and decisions.

An *integral* of the performance across a noise parameter space NPS with respect to a measure of noise should be used. An integral of a function f :NPS $\rightarrow [0, 1]$ with respect to an uncertainty measure $g : \mathcal{B} \rightarrow [0, 1]$ over a sigma-algebra of sets $\mathcal{B}$,

$$\int_N f(n)\, dg : (\mathrm{NPS}, \mathcal{B}, g) \rightarrow [0, 1], \tag{5}$$

is a map used to determine the performance rank of the function f over a subset N of the NPS as measured by the uncertainty measure g. This definition is quite general. For product design, there are specific restrictions the operation must satisfy, shown in Table 2, as developed in [5]. Any operation on an NPS that satisfies these conditions will be called an *uncertainty integral*.

A particular uncertainty form can be used when the events are random. For example, inaccuracies in measurements and manufacturing are usually modeled as random with a probability measure Pr. The overall

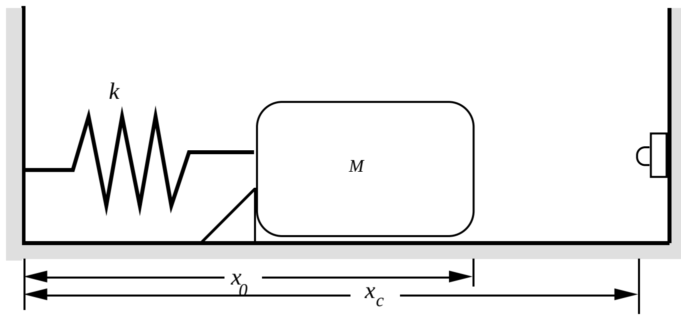


Figure 1: Example: Accelerometer configuration.

expected preferential performance at any design configuration becomes the integral of performance across the probability space.

$$\mu(d) = \int_{NPS} \mathcal{P}(\mu_1, \ldots, \mu_N)\, dPr \tag{6}$$

This integral is the standard Lebesgue integral from measure theory [3].

Product development also encompassed decisions which are not made by a designer (and so are not controllable by the designer) and which are not random. A possibility measure Π can be used to represent such variables. This will be shown in the accelerometer design example below, which has a backstop positioning adjustment, whose value is determined not by the designer, nor by randomness. For these uncontrollable variables, the overall expected preferential performance is evaluated across a possibility space.

$$\mu(d) = \sup \{\min\{\mathcal{P}(\mu_1, \ldots, \mu_N), \Pi(N)\} \mid N \in \{N_j\} \text{ disjoint} \subset \mathcal{B}\} \tag{7}$$

This integral is the Sugeno integral of possibility theory [7], expressed for when the NPS has the structure of a σ-algebra, as assumed for design. Note the *sup* is across the subsets of the partition $\{N_j\}$, and the limit as the partition becomes finer in $\mathcal{B}$ is used.

A final uncontrollable variable form represents uncertainty where proper performance even under the worst case must be ensured. For example, the effects of a noise variable to be contained within 3 standard deviations of a probability distribution. This can be modeled with a *necessity measure*, different from that developed by Dubois and Prade [1]. Each necessary noise variable has a subset that a designer feels must be ensured: if the noise variable takes on a value outside the subset, then the design will not function properly. This subset $\mathcal{N}_\alpha \in \mathcal{B}$ will be called the *necessary set*. Given this, any subset in $\mathcal{B}$ can be measured as necessary $\mathfrak{N}(N)$. If it lies within $\mathcal{N}$, then the set is necessary, otherwise it is not. This defines a crisp two-placed $\{0, 1\}$ *necessity measure*. The actual uncertainty is incorporated into the extent of $\mathcal{N}$ in the NPS. The overall expected preferential performance now becomes the integral of performance across the necessity space.

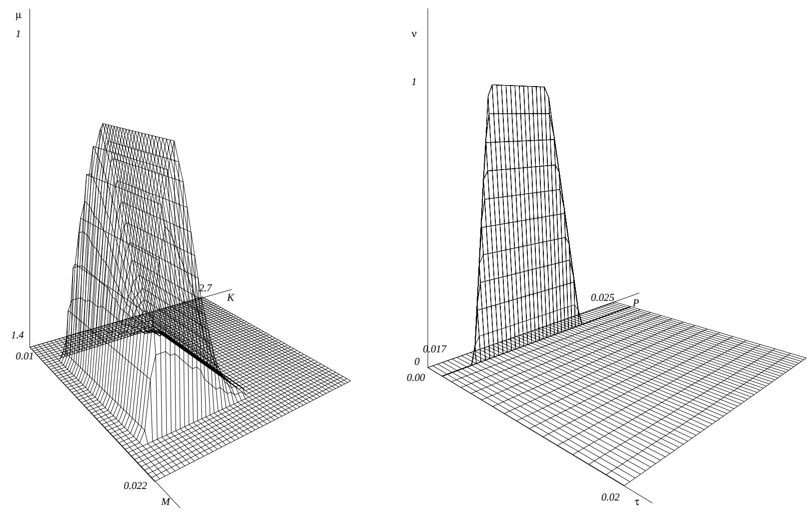
$$\mu(d) = \inf \{\max\{\mathcal{P}(\mu_1, \ldots, \mu_N), \mathfrak{N}_\alpha(N)\} \mid N \in \{N_j\} \text{ disjoint} \in \mathcal{B}\} \tag{8}$$

This integral of performance provides the worst case performance across the necessity space, as measured by the necessity measure. It can be verified that this definition is indeed an uncertainty integral.

Iteration in Design

The design paradigm presented so far assumes the preliminary design process can be carried out in a forward manner. Of course, there will be iteration among the steps presented. Observations in the achievable performance will induce changes in the specified preferences for design parameter values.

To illustrate our approach, we introduce an example below. The problem is to design an accelerometer that closes a switch when a threshold acceleration has been exceeded. One alternative configuration is shown in Figure 1. For this configuration, we wish to evaluate its feasibility as a design solution (even though it is



Overall Expected Preference   Induced preference

Figure 2: Accelerometer preferences.

imprecisely described), and also to suggest more precise values for the various parameters that describe this design.

The designer must select a mass (M) and spring constant (K), given a random variation δk on K. The performance of this design will be evaluated on the basis of the time to close the switch under a specified acceleration (τ), and the preload of the proof-mass against its stop (to reject spurious switch closures) (P). The design and performance parameters have associated preferences. The random variation has an associated density function. To assist in maintaining the targets on the goals, the manufacturing procedure can position the backstop (x_0) based on measurements made of the total spring constant. The backstop will have an associated possibility distribution. Applying (6) and (7) in conjunction with the precedence relation, the problem becomes to solve for M^*, K^* such that

$$\mu(M^*, K^*) = \begin{array}{l} \sup \left\{ \int_{\delta k} \sup \left\{ \min\{\mathcal{P}\{\mu(k), \mu(M), \mu(\tau), \mu(P)\}, \pi(x_0)\} \mid x_0 \in \mathbb{R} \right\} \right. \\ \left. \times pdf(\delta k)\ d(\delta k) \mid (M, K) \in \mathbb{R}^2 \right\} \end{array} \tag{9}$$

Notice that this rather typical design problem requires imprecision, probability and possibility in its formulation. The preference surface is graphed in Figure 2, the supremum of which is the most preferred point. This surface reflects the designer's expected preferences, given the manufacturing errors and the ability to overcome them to within the possibility of the adjustment.

The designer may initially specify only provisional preferences. To iteratively refine her preferences, she can induce the preferences specified on M and K, and propagate them onto the performance variables (using the *forward* calculations discussed above). This is shown in Figure 2, and as can be seen, only a small segment of actuation times are available with the design configuration values initially desired. This understanding can be used to further refine the design.

Suppose now the designer has performed analysis described and is prepared to decide which final design parameter values to use. That is, the designer wishes to combine all the preferences into an overall preference. We have shown [6] that points which maximize overall preference can also be determined by another method. One can induce the preferences specified on the DPS $\mu(d)$ onto the PPS by the imprecision transformation to create $\nu(p)$, in a *forward* manner. There one can find the $p \in$ PPS that maximize an overall preference, and then back map to the DPS simply by looking up the values of d used in the original forward mapping ν at the optimal p. This is true even though the induced preference ν on the PPS involves only the preference $\mu(d)$, and does not consider any of the dependent set preferences $\mu(p)$. The results, however, are the points which maximize the overall preference μ_c : the supremum of the combination of the preferences specified both on DPS and PPS.

After having made the induced preference calculations, the designer can observe the imprecise performance achievable and proceed to judge the candidates. Of course, this process can continue even when the designer has preliminary preferences specified on all the parameters. Given any parameter, the induced preference from the other parameters can be calculated, and the ν and μ can be compared. In doing so, any specification μ or $\mathcal{P}$ can be modified, and the resulting ν observed. Finally, when the overall model is accepted, the backwards path can be used to determine the overall most preferred points in the DPS.

This technique is superior to others, since it presents visual information to the designer about the model. An optimization routine will produce a (hopefully globally optimal) solution, but it presents little information about critical aspects in the problem. Further, computational concerns are attenuated since the induced preference pre-image of the optimal performance value is the optimal preference point, thereby eliminating the need for a subsequent search.

Conclusion

Imprecise information is a necessary and intrinsic part of engineering design. However, imprecision in engineering design is often overlooked, and rarely formalized. Our method, utilizing the mathematics of fuzzy sets, represents and manipulates imprecision in engineering design. This includes the effects of customer and designer preferences, formal strategies for trade-offs (including the ability to record these trade-off decisions for later examination or modification), iteration in engineering design, and noise (such as uncontrollable manufacturing and material property variations).

While other methods, such as utility theory, exist for similar purposes, they do not capture designers and customers preferences, nor do they permit engineering trade-offs using a variety of strategies. A comparison of our method with utility theory is more fully discussed in [4].

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