Surface Reconstruction of Etched Contours

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ABSTRACT

Previously, a computationally efficient and geometrically accurate etching simulation was developed to compute contours of constant depth at different time steps [1]. Experimental verification of this early work has been established [2]. However, contours are not amenable to the analysis and design of microsystems, since it is often necessary to determine object interaction and mechanics in which surface or solid representations of shape are required. Hence, the problem of surface reconstruction is of particular interest. It has been noted that surface reconstruction is an interpolation problem, explaining the predominance of triangulation techniques in many surface reconstruction algorithms [3], [4]. One very effective code, by Boissonnat and others, partitions the contours in a single cross section into Delaunay triangles, then connects the triangles between adjacent cross sections to form Delaunay tetrahedra that can be used for finite element analysis. The surface representation is easily extracted from the bounding tetrahedra. An implementation of Boissonnat's algorithm to the problem of surface reconstruction of etched contours is discussed. Further, a simple algorithm is presented to remove the bounding triangulations created by Boissonnat's code (*i.e.*, the surfaces are closed). An application written in TCL and VTK, a freely available highly portable graphical toolkit, to interactively view the resulting surfaces of triangles is discussed.

Keywords: MEMS, surface reconstruction, bulk etching, etching simulation

INTRODUCTION

In an earlier work, a computationally efficient and geometrically accurate etching simulation was reported that computed etched contours of constant depth at multiple time steps [1]. Experimental verification of this work has been established [2]. However, contours are not amenable to the analysis and design of microsystems, since it is often necessary to determine object interaction and mechanics in which surface or solid representations of shape are required. Hence, the problem of surface reconstruction is of particular interest.

Typically, for bulk-etching there can be more than one closed contour per cross section and enclosed contours as well. A useful surface reconstruction algorithm must then be able to handle multiple contours per depth level. Current surface reconstruction algorithms can be dichotimized into voxel (3D pixels) based approaches and triangulation based approaches. Both approaches have been used to develop algorithms that accurately reconstruct surfaces from multiple contours on multiple cross sections. Voxel based approaches require large amounts of memory, since a three-dimensional rectangular grid must be constructed from the data set. In addition, voxel techniques require filled contours rather than contour lines/boundaries. Triangulation based approaches only require vertices of contour boundaries and a winding direction, resulting in faster execution when compared to voxel techniques. One such triangulation algorithm by Boissonnat was used to reconstruct surfaces of etched contours obtained from the previously reported etching simulation. It should be noted that much of the surface reconstruction research was done for bio-medical imaging purposes (*i.e.*, surface reconstruction of CAT scans).

SURFACE RECONSTRUCTION VIA DELAUNAY TRIANGULATION

Boissonnat developed a very robust and efficient algorithm for surface reconstruction based on Delaunay triangulation. Several geometrical concepts need to be clarified and expounded before further discussion of the algorithm. A less formal approach to the presentation of these concepts is taken here than those in computational geometry, in light of space constraints.

Delaunay Triangles and Medial Axes

Given a point set $\boldsymbol{V}$, an edge connecting any two points in $\boldsymbol{V}$ is a Delaunay edge if and only if there exists an empty circle that passes between the two points. A *Delaunay triangle* is then defined to be a triangle whose edges are all Delaunay. The center of the circle bounding the Delaunay triangle is called the *circumcircle*. Furthermore, it can be proven that a Delaunay triangulation is unique. There are many methods for Delaunay triangulation of points sets and the interested reader is referred to [5]. Application of Delaunay triangles to the third dimension leads to Delaunay tetrahedra. While Delaunay triangles and tetrahedra do not necessarily minimize edge lengths, in general, Delaunay triangulation connects closest neighbors [6].

The *medial axis* of a polygon $\boldsymbol{P}$ is the locus σ of its internal points such that each point $p \in \sigma$ is equidistant from at least two points on the boundary of ***P*** [6]. The medial axes can also be thought of as the locus of internal points at which fires started at the polygon boundaries meet. Likewise, an external medial axes is the locus of points external to two or more polygons such that each p is equidistant from at least two points on the boundaries of the polygons.

Surface Reconstruction Using Delaunay Triangulation

Boissonnat's surface reconstruction algorithm can be divided into two parts. In the first, contours on each cross section are triangulated and medial axes are computed. Then, contours on adjacent cross sections are joined to create a surface using the previously computed triangulations and external medial axes. Here, a simplified description of the algorithm is given.

It should be noted that 2D triangulations of contours often require addition of new vertices to ensure that all contour edges remain after the computation. This is called a constrained Delaunay triangulation, in which specified edges must still be present after triangulation has been completed. Tetrahedra are formed by connecting a 2D triangle from one cross section to the vertex on the adjacent cross section closest to the triangle's circumcircle. In [3] Boissonnat shows that these tetrahedra are Delaunay, such that nearest neighbors are most likely connected. However, new vertices may need to be inserted to handle changing number of contours from cross section to cross section. As an example, Figure 1 shows a comparison of surfaces constructed with and without insertion of a strategically placed vertex. The location of such vertices are determined by projecting the external medial axes onto a single plane and finding the internal intersections. This partitions the contours such that their vertices are connected to their nearest neighbors on the adjacent cross section, due to properties of the medial axes (Figure 2). The algorithm is implemented in C code and can be obtained at *ftp://ftp-sop.inria.fr/prisme/NUAGES/Nuages*; the program is called *Nuages*.

Thus, Boissonnat's surface reconstruction algorithm [7] can be enumerated as:

1. In each cross section:

   (a) Compute the 2D Delaunay triangulation of the vertices

   (b) Add vertices to satisfy constrained Delaunay conditions (*i.e.*, all edges remain after the triangulation)

2. With each pair of adjacent cross sections:

   (a) Extend the two 2D triangulations to one 3D Delaunay triangulation

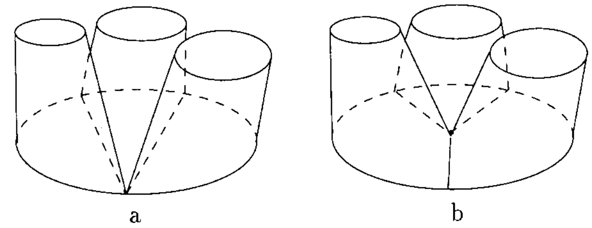

Figure 1: Comparing reconstructed surface (a) without and (b) with added vertex. *Source:* J.D. Boissonat, Three dimensional reconstruction of complex shapes based on the delaunay triangulation. *INRIA Research Reports*, 1697, April 1992.

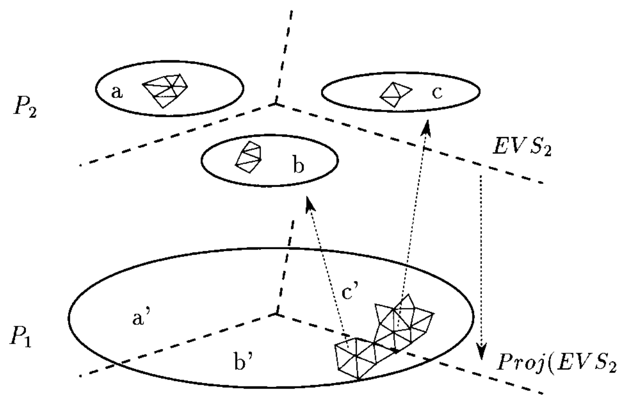

Figure 2: Showing how new vertices are determined. *Source:* J.D. Boissonat, Three dimensional reconstruction of complex shapes based on the delaunay triangulation. *INRIA Research Reports*, 1697, April 1992.

RESULTS

Nuages is used in conjunction with a highly portable graphical toolkit called *VTK* (Visualization Toolkit) to reconstruct surfaces of bulk-etched contours. An interactive program was developed using TCL/TK and *VTK* to view the reconstructed surfaces. A screen shot of the viewing application is shown in Figure 3. Figures 4 and 5 show representative contours and their reconstructed surfaces. Surfaces reconstructed from multiple contours often have noticeable indentations, most likely due to the insertion of new vertices in the constrained triangulation step.

The intended use of *Nuages* was for reconstruction of MRI, CT, *etc.*, data of organs. Hence, computed surfaces are necessarily closed. Bulk-etching, on the other hand, typically involves open surfaces. By noting that the triangulation of any convex polygon with n vertices is n-2 triangles, the capped (or top most) surface can be easily removed. On occasion, a few triangles in the capped surface will remain since contours are not always convex.

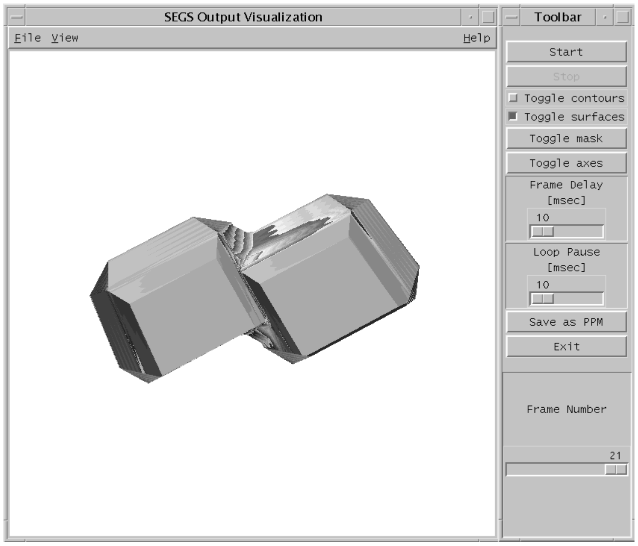

Figure 3: Screen shot of etching simulation viewer.

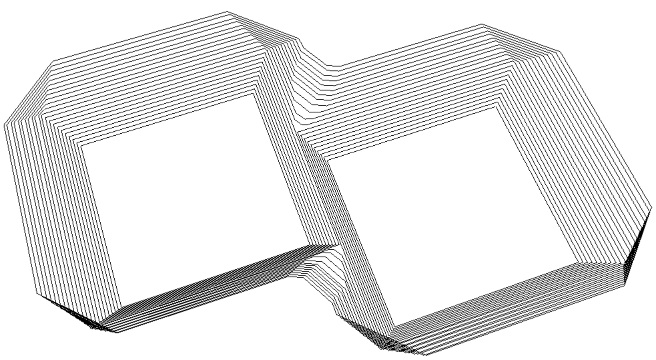

Figure 4: Contours from Etching Simulation.

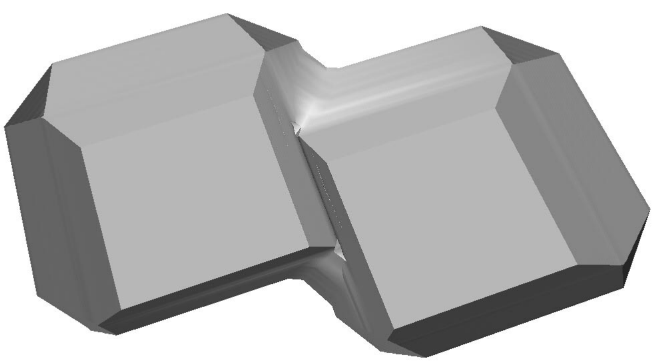

Figure 5: Surface Reconstruction of Contours from Figure 4.

FUTURE WORK

Currently, work is being done to integrate the bulk-etching simulation, surface reconstruction via *Nuages*, and interactive viewing application to build a etching simulator package.

A new algorithm is being examined in which only constrained 2D Delaunay triangulations are required for surface reconstruction. For bulk-etching, it can be shown that contours on adjacent cross sections will not intersect. By projecting adjacent cross sections onto the same plane, a 2D triangulation can be made, connecting the contours. A 3D surface can then be extracted by returning each point's original ”z-dimension”. Such an algorithm will result in large time savings since the 2D to 3D triangle extensions are no longer required.

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